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Ancient
VEDIC MATHEMATICS

PUSHP–2

(A Magic till un ierstood)

वैदिक गणित सूत्र :

१. एकाधिकेन पूर्वेण
२. निखिलं नवतश्चरमं दशतः
३. ऊर्ध्वतिर्यग्भ्याम्
४. परावर्त्य योजयेत्
५. शून्यं साम्यसमुच्चये
६. (आनुरूप्ये) शून्यमन्यत्
७. संकलनव्यवकलनाभ्याम्
८. पूरणापूरणाभ्याम्
९. चलनकलनाभ्याम्
१०. यावदूनम्
११. व्यष्टिसमष्टिः
१२. शेषाण्यङ्केन चरमेण
१३. सोपान्त्यद्वयमन्त्यम्
१४. एकन्यूनेन पूर्वेण
१५. गुणितसमुच्चयः
१६. गुणकसमुच्चयः

By :

DR. NARINDER PURI

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वेदात् सर्वं प्रसिद्धयति ।

वेद सम्पूर्ण ज्ञान के अनादि अनन्त स्रोत हैं ।

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गुरुकुल कांगड़ी विश्वविद्यालय
कृपया पुस्तक के ऊपर कोई निशान आदि
न लगायें।

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गुरुकुल कांगड़ी विश्वविद्यालय, हरिद्वार
वर्ग संख्या 510 आगत संख्या 5959
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पुस्तक—वितरण की तिथि नीचे अंकित है। इस तिथि सहित २० वें दिन तक यह पुस्तक पुस्तकालय में वापिस आ जानी चाहिए। अन्यथा १० पैसे के हिसाब से विलम्ब-दण्ड लगेगा।

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ANCIENT VEDIC MATHEMATICS

Magic Speed Answers
to all Mathematical Problems using

SIXTEEN SIMPLE SUTRAS FROM THE VEDAS

5959

DONATION

PUSHP-2

By

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Pushp-2
Ist Edition
National youth day
Swami Vivekananda's Birthday
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Dr. Narinder Puri

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प्रातः प्रभृति सायान्तम्
सायन्तः प्रातदन्तम्
यत् करोमि जगन्नाथ
तदस्तु तव पूजनम्

सुबह से शाम तक, शाम से सुबह तक,
मैं जो कुछ भी करता हूँ, हे जगन्नाथ
सब तेरी ही पूजा, तेरी ही उपासना हो जाए।

Dedicated to

Jagad Guru Shankara Charya
Swami Bharati Krishen Tirathji Maharaj.

★ ● ★ ● ★ ● ★ ● ★

Vedic Mathematics provides very easy, one line, mental and superfast methods alongwith magic speed cross checking system As such Vedic Maths is a boon for all competitions, Vedic methods are easy to understand, and their variety, speed and easebrings joy in the heart and smile on the face of the students. The Vedic Maths always provides a general method which is always faster than the present system but Vedic system also provides various other methods which often work with magic speed and very frequently the students of Vedic maths class move faster than the teacher and speak out the answer before they are written on the black board.

Prof. K. R. Williams of London says, 'This element of choice in the Vedic System, even of innovation, together with mental approach, brings a new dimension to the study and practice of Mathematics. The variety and simplicity of methods brings fun and amusement, the mental practice leads to a more agile, alert and intelligent mind and innovation naturally follows'.

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SWAMI BHARATI KRISHNA TIRTHA

Swami Bharati Krishna Tirtha, author of the first recent Vedic Mathematics book was the Shankaracharya of Goverdhan Peath. Jagadguruji, named Venkatraman in his early days, was an exceptionally brilliant student and invariably won the first place in all the subjects in all the classes throughout his educational carrier.

After winning the highest place in the B A. examination, Shri Venkataraman Saraswati appeared at the M. A. examination of the American College of Sciences, Rochester, New York from Bombay centre in 1903. In 1904, at the age of just twenty, he passed M A, examination in further seven subjects simultaneously securing the highest honours in all, which is perhaps the all time world-record of academic brilliance. His subjects included Sanskrit, Philosophy, English, Mathematics, History and Science.

In the words of Swamiji,

"The Sutras (aphorisms) apply to and cover each and every chapter of each and every branch of mathematics (including arithmetic, algebra, geometry—plane and solid, trigonometry—plane and spherical, conics—geometrical and analytical, astronomy, calculus etc. etc.). In fact, there is no part of mathematics, pure or applied, which is beyond their jurisdiction. The Sutras are easy to understand, easy to apply and easy to remember; and the whole work can be truthfully summarised in one word "mental";".

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Preface

Vedas are the source of all knowledge and of all Sciences. They are not only the oldest but also the best. The Vedas have guided millions of aspirants on the path of knowledge. With the passage of time most of the Vedic knowledge has been scattered and shattered. Vedic Mathematics is no exception to it. It is inspiring to observe that, deep interest is being taken, the world over in reviving the Vedic Sciences.

There are very few persons who are experts in so many facets of knowledge Srimat Swami Bhartiya Krishna Tirth ji was a personality of that rare talent. Touched by the remark A mistake in Vedas'—about the crisp 'Ganit Sutras' in Vedas; he resorted to the age old Vedic method of meditation. He authored 16 volumes, but only the rewritten introductory volume is available, to inspire and guide the real seekers of knowledge. My humble pranams at his holy feet.

Every body who has learnt a little about the Ancient Vedic Mathematics has shown a great enthusiasm to learn more. The interest and response of the students have inspired us to link-up the available bits here and there.

The real beauty of the Ancient Vedic Mathematics is that as you start using this system, new links automatically come to the mind. A large number of persons, mostly students, from remote corners of mother India have helped me in reviving the new links. (The chapters 1, 2, 3, 4, 7, 8 & 11 are beyond the basic book).

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We are trying to put the already revived links in a serialised form which may be easily learnt by the students and teachers alike. The write-up has become rather long, at places, even for simple topics. But once learnt and practised, the actual operations can frequently be performed with a superfast speed The limitations and criticism may kindly be freely conveyed, for improvement in future.

This whole project is being done with the spirit of 'Jnyana Yajna'. Thanks are due to a large number of selfless workers for their labour of Love ! I would like to specially thank Kumari Pragati Goel for her unique help. Thanks are due to Sh. R. S. Tyagi ji.

Meenakshi has always inspired me.

The response from all parts is encouraging us for further work We are glad to see that the Government of India is taking a keen interest in this, and we hope Vedic Maths shall soon find its due place in education.

May the students enjoy this Ancient Vedic gift, which makes mathematics a fun, rather than a burden. —N. Puri

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'ANCIENT VEDIC MATHEMATICS'

SALIENT FEATURES

A Complete System

- 16 Vedic sutras of 120 words can solve all mathematical Problems.
- With a firm scientific footing
- A revolution in concept of learning
- Opening new areas of R & D
- The Vedic Maths can be used to check the Computer results, and frequently supersedes calculators
- Revealing hither-to unknown, Great and Mysterious Qualities of Numbers
- Simple, short mental mathematical processes often with magic speed
- Requires no prerequisite courses
- Mathematics otherwise burden, turns to be a joyful subject
- A boon for all competitions like Banking, Engineering, Medical, Economics UPSC, ONGC, MBA, etc., where the use of calculator is not allowed
- More useful, accurate and applicable at times, compared to modern computers
- 2 hours/day for 11 months equals 12 years of normal course in mathematics in India and abroad.

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VEDIC MATHEMATICS NEWS

"It is tempting to think of them (Indians) as computer scientists.....Let us not forget that among the great accomplishments of the Indian thinkers were the invention of zero, and of the Binary number system a thousand years before the West". Dr R Briggs of NASA, California, in Jl. of Artificial Intelligence, Spring, 85.

It is interesting to note that this Ancient Indian Binary number system is used in all the calculators and computers of the world without exception.

1. The first book on Vedic Mathematics was written by H. H. Swami Bharati Krishen Tirth Ji, Shankracharya of Puri (1884-1960) in English This book is already running in 10th reprint in last 20 years. It has already been translated in Marathi in 4 volumes and Gujrati in 3 volumes. Further, it is being translated in Hindi, Latin & German.

2. The Symposium and Workshop have been organised at Nagpur, London and Roorkee in 1985, 1986 and 1987 respectively.

3. The 'Vedic Mathematics' is being taught at the School of Economic Science London, James School, St. Vedast School London, Mary Ward Centre, School of Philosophy Australia and Auckland, and several other institutions in USA Australia and Holand etc.

4 Swami Ji's book introduces the wonderful applications to Arithmetical computations, theory of numbers, compound multiplications, algebraic operations, factorisations, simple quadratic and higher order equations

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simultaneous quadratic equations partial fractions, check squaring, cubing, square root, cube root, coordinate geometry and the wonderful Vedic Alpha-Numerical code.

5. Currently a course is being run in London demonstrating that 'Vedic Methods' can be used for G. C. E. A-level mathematics. This is a pilot project, being held at the Mary Ward Centre, in association with the Polytechnic of North London This is equivalent to our 10+2 Level.

6. On this novel Indian system of Mathematics pioneering work is being done in U K. The research includes applications of Vedic Mathematics to trignometry, three dimensional coordinate geometry, solution of plain and spherical triangles, linear and non-linear differential equations, matrices and determinants, and log and exponentials.

7. The most interesting point to note is that the Vedic Mathematics provides unique solution in several instances where only trial and error method is available at present. For example the Kepler's equation can be solved in just 90 seconds as comparad to the present tedious trial and error method, which takes nearly half an hour.

LIST OF NEW BOOKS ON 'VEDIC MATHEMATICS'

1. 'Introductory Lectures on Vedic Mathematics', by A P. Nicholas, J. R Pickles and K William.
2. 'Discover Vedic Mathematics', By K. William.
3. 'Triples', by K. Williams. (A research book)
4. 'Vertically and Cross-wise', by A. P. Nicholas, K Williams, and J. R. Pickles.
5. 'Vedic Ganit' by Dilip Kulkarni in Marathi, 4 parts.
6. Vedic Ganit in Gujrati, 3 Parts.
7. Vedic Ganitam, by T. S. Raju.

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8 Notes of Workshop on "Vedic Mathematics and its Computational Potentialities" based on advanced research in U. K., G Editor N. Puri (English).

9. Ancient Vedic Mathematics—Pushp-1 by N Puri (Hindi/English). The only book available in Hindi.

10. Maths or Magic by J House (1976)

11 Glimpses—Vedic Maths, by N Puri [Hindi/English].

Note :—All books are available from SSG. Send Rs. 2/- for full details.

WORK AT ROORKEE

The Spiritual Study Group consisting of the students and staff of Roorkee University has initiated humble efforts towards the research, developement and spreading of this wonderful mathematical facet of our rich, vast & scientific cultural heritage.

Video Cassettes : A series of Video Cassettes of Lectures on different facets of Vedic Mathematics are being produced. These are available on sale or hire bases The interested persons or institutions may send Rs. 10/- for full details of contents of each cassette, rules and regulations of hire/purchase system etc.

LECTURES :

A large number of Vedic Mathematics demonstration lectures have been delivered at hundreds of schools, colleges and Universities in Delhi, Punjab, Haryana, Gujrat, W. Bengal, UP., MP, Kerala, Goa and Maharashtra.

COURSES :

On repeated requests since May 85, SSG has already organised the VM courses for 31 batches, attended by

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1750 participants, ranging from school children, teachers, university students, research scholars, field engineers to the professors of the university, by Jan., 1988. All of them enjoyed learning the simple, magic speed methods of solving the otherwise tedious mathematical calculations.

Institute of Vedic Sciences-Correspondance course

On repeated requests from persons residing outside Roorkee, the correspondence course for home study of the wonderful and magic speed "Vedic Mathematics" has been started from the auspicious Maha Kumbh, Baisakhi (April 14, 1986). With a view to revive the ancient sciences, SSG has started the Institute of Vedic sciences at Roorkee. Presently the Institute is organising Correspondence courses of Six months duration in Hindi and English. New batches start from auspicious Baisakhi (April) and Dushera (October) every year.

Any body who is in class VIII and above can join these courses. Fortnightly lessons, containing a large number of solved problems, and examples are sent to every participant. The students are asked to submit solution of the problems, which are duly checked. A certificate is awarded at the end of the course, after a postal test. The detailed information, course content, glimpses of Vedic Maths. etc are available for Rs. 10/- (further add. Rs 5.00 if desired by registered post).

4. Workshop :

A Symposium on Vedic Mathematics was held in Feb. 1985 at Nagpur. The second at Bhartiya Vidya Bhawan's London Centre in Sept 1986. Recently the first 11-day Workshop on Vedic Maths. and its computational potentialities was held at University of Roorkee in Aug.

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1987 84 participants joined the workshop. Advanced research level topics like inversion of matrices; solution of non-linear differential equations, trig functions etc. were handled. The special feature of this workshop was a series of 15 lectures by Prof. A P. Nicholas of Polytechnic of North London. The entire workshop meterial is available for sale. The video cassettes have been recorded

5. Ancient Vedic Mathematics :

With a view to provide the comprehensive details of this wonderful, ancient Bhartiya Cultural Heritage, SSS is planning to publish a series of books called Pushp-mala.

S.S S. most prayerfully and humbly publishes this Pushp-2 in English Pushp-3 shall be published by mid 88.

Ancient Vedic Mathematics New Links

As a result of the humble research efforts of the members of SSG, a few of the missing links of the Ancient Vedic Mathematics have already been revived, Specifically the superfast Vedic methods of addition, subtraction, compound addition and subtraction, Tables, compound Multiplication, extensions of the Base System for multiplication multiplication of three or more numbers in one line, squaring of very large numbers, the magic speed methods of checking the correctness of any arithmetic or algebric addition, subtraction, multiplication, division, factorisation, etc. have already been re-established. Many of the new links alongwith multiplication, division, fractions etc. are being published in Pushp-1 & 2. Further links shall be published in future Pushp mala.

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5 Opinions of the Participants :

Everybody who has learnt a little about the Vedic Mathematics has been thrilled by the superfast, almost magic speed calculation procedures of the Vedic Mathematics. The Spiritual Study Group at Roorkee has been organising Elementary and Advance Courses on Vedic Mathematics, since May, 85. All the participants are asked to fill up the response forms comparing the present system of Mathematics with the Vedic system. A total of 11 questions were asked. The minimum score in favour of Vedic Mathematics for any question in any course sofar was 80%. Most of the questions got 100% favour. Further, comments of participants are motivating and inspiring for spreading Vedic Maths.

A few glimpses of the participants opinions :

— A boon for all competitive examinations.

— Could never imagine it would be so interesting.

— Very pleasant to have this course at the university.

— How advanced our ancestors were even in Mathematics.

— If Vedic Maths was taught in every school, computers would have developed in India.

— Excellent course.

— Want to learn advanced topics.

— Increases devotion towards our INDIA and its ancient culture.

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— Now I wish to learn Sanskrit to know more about our ancient scientific developments.

— Every person on earth should have the knowledge of Vedic Mathematics

— It was a boon to learn Vedic Mathematics in Dushera holidays (I had planned for my regular studies).

— Want to learn more about Vedic Mathematics like factorisation etc.,which are difficult in modern system.

— Amazing speed, very wide applications covering almost all the mathematical operations

— A dynamic step for its introduction in educational system is a must.

— Has created an interest to know more about it

— It can be of great use for children if made part of syllabus.

— Mysterious nature of numbers. After little practice problems can be mentally solved at magic speed.

— Very educative, has much scope for coming generation to start its practice from beginning.

— It is a strong proof of our rich cultural heritage.

— By Vedic methods maths, which is a burdon, becomes a joy for the children.

— This is the most interesting and thrilling time of my life Being a mathematics teacher I have interest in maths, but this Vedic Mathematics has thrilled me. When I come in contact with the Vedic Mathematics it gave such a pleasure which I can't explain in words.

A teacher of University Adarsh Bal Niketan

— The lectures were eye openers and most thrilling.

— A glimpse of the lofty trakking in Himalayas of ancient knowledge in Maths, developed with spiritualism. Cudos to SSG.

A Professor of University

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ॐ तेजोऽसि तेजोमयि धेहि
वीर्यमसि वीर्यमयि धेहि
ओजोऽसि ओजोमयि धेहि
बलमसि बलमयि धेहि
अविरावीमयी ।

हे तेज के पुन्ज हमें तेज दो ।
हे वीर्य के पुन्ज हमें वीर्यवान बनाओ
हे ओजस्व स्वरूप हमें ओजस्व दो
हे बल के पुन्ज हमें बल प्रदान करो
हम अपने स्वरूप की ओर अग्रसर करें ।

★ ★ ★ ★ ★ ★

ॐ सहनाववतु सहनौ भुनक्तु सह वीर्यं करवावहै ।
तेजस्विनावधीतमस्तु मा विद्विषावहै ।
ॐ शान्तिः शान्तिः शान्तिः ।

हे परमात्मा हम गुरू और शिष्य दोनों की रक्षा करो । हम दोनों का पालन करो । हम दोनों साथ में रहकर तेजस्वी दैवी कार्य करें । हम दोनों का किया हुआ अध्ययन तेजस्वी दैवी हो और हम दोनों परस्पर एक दूसरे का द्वेष न करें ।

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CHAPTER 1

VINCULUM OPERATIONS

1·1· Introduction :

In Pushp 1 we have seen how we can use the Nikhilam Sutra (निखिलं नवतश्चरमं दशत:) and reduce the bigger digits of any number to be at the most equal to 5. This Vinculum (विनकुलम्) form of numbers is a unique facet of Vedic Mathematics Through this process the computations become far more easier than otherwise. As a matter of fact the unique concept of Vinculum in the Vedic mathematics is a very interesting and useful device.

1·1·1 The Approach :

In the Vinculum process we take the complement of the digits bigger than 5 with a negative sign, and a positive carry over to the next higher order digit.

(refer Chapter 3 Pushp 1)

1·1·2 Illustration 1 : $79 = 80 - 1 = 8\bar{1}$

Further $80 = 100 - 20 = 1\bar{2}\bar{0}$

therefore $79 = 8\bar{1} = 1\bar{2}\bar{1}$

So we find 79 can be written as $8\bar{1}$ but since the digit at the tenth place is also very big we can take its complement (from 10, 10−8=2) and increase the digit previous to this by 1. (0+1=1, therefore 79 becomes $1\bar{2}\bar{1}$)

It is interesting to note that if there are a few digits larger than 5 appearing together, their collective complement can be directly written by using the Nikhilam Sutra

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and increasing the previous digit by 1. In this case 9—7=2, 10—9=1, giving the complement as 21 and the previous digit 0 becomes 0+1=1. Hence $79=1\bar{2}\bar{1}$.

1·1·3 Illustration 2 : 286923

The complement of the larger digits (869) by Nikhilam Sutra is 131. Therefore

$286923=3\bar{1}\bar{3}\bar{1}23$

Note that the last digits 2 and 3 at the tenth and the unit place stay on as before, and the digit previous to the bigger ones gets increased by 1.

1·1·4 Illustration 3 : 782893

$782893=1\bar{2}\bar{2}3\bar{1}\bar{1}3$

1·1·5 Normalizing : Let us further consider a few examples of normalizing the Vinculum number. It shall be surprisingly interesting to learn that the Nikhilam sutra can be used even for the normalizing process. The only difference being that the previous digit is now reduced by 1.

1·1·6 Illustration 4 : $3\bar{3}\bar{4}\bar{2}$

Since there are three digits in a group we can use the Nikhilam sutra to straight away write down the complement of this group, and we should remember to reduce the digit previous to this group by 1.

Therefore, by Nikhilam 9—3=6, 9—4=5 and 10—2=8, and 3—1=2.

As such $3\bar{3}\bar{4}\bar{2}=2658$

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1·1·7 Illustration 5 : $2\bar{3}\bar{2}6\bar{1}\bar{1}2$

Therefore,

$$2\bar{3}\bar{2}6\bar{1}\bar{1}2 = 1685892$$

1·2 Addition of Vinculum Numbers (Part I) :

The addition of numbers in Vinculum form is similar to the ordinary addition the only difference being that each digit is operated alongwith its sign. The following examples shall explain the point.

1·2·1 Illustration 6 : We want to add

$$\begin{array}{r} 2\ \bar{3}\ 4\ \bar{2} \\ +\ 4\ \bar{2}\ \bar{1}\ 3 \\ \hline \end{array}$$

Step. 1 : At the unit place 3 is to be added to $\bar{2}$ resulting in $3+\bar{2}=3-2=1$.

Step. 2 : $\bar{1}$ is to be added to 4 at the tenth place therefore $4+\bar{1}=4-1=3$.

Step 3 : $\bar{2}$ is to be added to $\bar{3}$ at the hundredth place giving us $\bar{2}+\bar{3}=-2-3=-5=\bar{5}$.

Step 4 : 4 is added to 2 at the thousandth place. This gives $4+2=6$. Therefore we get

$$\begin{array}{r} 2\ \bar{3}\ 4\ \bar{2} \\ +\ 4\ \bar{2}\ \bar{1}\ 3 \\ \hline 6\ \bar{5}\ 3\ 1 \\ \hline \end{array} \quad = 5531$$

1·2 2 Illustration 7 :

$$\begin{array}{r} 3\ \bar{4}\ \bar{4}\ 2\ \bar{2} \\ +\ 7\ \bar{5}\ 3\ \bar{5}\ 4 \\ \hline \end{array}$$

Step 1 : At unit place $4+\bar{2}=4-2=2$.

Step 2 : At tenth place $\bar{5}+2=-5+2=-3=\bar{3}$.

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Step 3 : $3+\bar{4}=3-4=-1=\bar{1}$ at hundredth place.
Step 4 : At thousandth place $\bar{4}+\bar{5}=\bar{9}$.
Step 5 : $7+3=10$ therefore we get $10\bar{9}\bar{1}32$

By Nikhilum sutra we write the complement of the Vinculum portion (0913) and reduce the previous digit by 1 $(1-1=0)=90872$.

1·3 Subtraction of Vinculum Numbers (Part I)

The subtraction of numbers in Vinculum form is similar to the ordinary subtraction, the only difference being that each digit is operated along with its sign. The following examples shall explain the point.

1·3·1 Illustration 8 :

1. $4-3=1$.
2. $3-4=-1=\bar{1}$.
3. $4-\bar{2}=4-(-2)=4+2=6$.
4. $\bar{3}-2=-3-2=-5=\bar{5}$.
5. $\bar{2}-\bar{3}=-2-(-3)=-2+3=1$.
6. $\bar{4}-\bar{2}=-4-(-2)=-4+2=-2=\bar{2}$.

The subtraction of the numbers in the Vinculum form shall consist of only one of the above six cases. The subtraction of individual digits can be carried out without any borrowing process as shown above in case 2, 4, 5 and 6.

1·3·2 Illustration 9 :

$$\begin{array}{r} 5\ \bar{4}\ 2\ \bar{3}\ 1 \\ -\ 2\ \bar{2}\ 3\ \bar{4}\ 3 \\ \hline 3\ \bar{2}\ \bar{1}\ 1\ \bar{2} \end{array}$$

Step 1 : $1-3=-2=\bar{2}$ at the unit place

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Step 2 : $\bar{3}-\bar{4}=-3-(\bar{4})=-3+4=1$ at tenth place.

Step 3 : $2-3=-1=\bar{1}$ at hundredth place.

Step 4 : $\bar{4}-\bar{2}=-4-(-2)=-4+2=-2=\bar{2}$ at thousandth place.

Step 5 : $5-2=3$ Therefore we get the answer as $3\bar{2}\bar{1}1\bar{2}$, which in the normalized form gives 27908.

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1·3 3 Illustration 10 :

$$\begin{array}{r} 3\ 7\ 1\ 0\ 8\ 2 \\ -\ 1\ 5\ 6\ 4\ \bar{1}\ \bar{1} \\ \hline 2\ 2\ \bar{5}\ \bar{4}\ 9\ 3 \end{array}$$

Step 1 : $2-\bar{1}=2-(-1)=2+1=3$ at unit place

Step 2 : $8-\bar{1}=8-(-1)=8+1=9$ at tenth place.

Step 3 : $0-4=\bar{4}$ at hundredth place, and similarly.

Step 4 : $1-6=-5=\bar{5}$.

Step 5 : $7-5=2$.

Step 6 : $3-1=2$ Therefore $22\bar{5}\bar{4}93=214693$.

1·4 Discussion :

1. The above examples illustrate the operations of simple digit by digit addition and subtraction of Vinculum numbers.

2. The usual borrowing process is not required even when the digit to be subtracted is bigger than the other digit.

3. In all above examples the carry over was not required. This however may not be always true. In case, the digits being added yield the sum larger than 9 the excess unity at the tenth place is carried to the next column through the simple shudhikaran process as explained in the chapter of addition (refer Pushp 1).

1.5 Multiplication of Vinculum Digits :

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The product of digits in the vinculum form is done by using the simple algebraic rules.

1·5.1 Illustration 11 :

1. $3 \times 2 = 6$.
2. $\bar{2} \times 4 = -2 \times 4 = -8 = \bar{8}$.
3. $\bar{3} \times \bar{2} = -3 \times (-2) = 6$
4. $2 \times \bar{3} = 2 \times (-3) = -6 = \bar{6}$.
5. $\bar{4} \times 3 = -4 \times 3 = -12 = \overline{12}$.
6. $4 \times \bar{5} = 4 \times (-5) = -20 = \overline{20}$.
7. $\bar{3} \times \bar{4} = -3 \times (-4) = 12$.

1·6 Discussions :

1. It is important to practice the Vinculum system a bit more as we are not familier with the same.

2. However, it is interesting to know that the Vedic Vinculum system though not taught, is being extensively used in computers.

3. Many of the operations become much simpler when the Vinculum system is used for calculation, as shall be explained in the following chapters and subsequent volumes.

1·7 Addition of Vinculum Numbers (Part II) :

In all the illustrations no carryover of any type was required. Generally the carryover shall not be required as most of the digits shall not be more than 5. However this may not be always true. Further it is important to be clear that all the digits of a number in vinculum form need not always be 5 or smaller. The Vinculum number essentially consists of both positive and negative digits,

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and as we already know, the negative digits are shown with the bar above them.

While adding the Vinculum numbers the carryover may be sometimes required. This is done in the usual Vedic style by using the Shudha Sutra (Refer chapter 4 Pushp 1). However in operation of Vinculum numbers two types of situation may arise. The first, when the addition of positive digits results in a number greater than 9 We say 'Shudha' and put a dot over the last digit just added, drop the unity at the tenth place and retain the digit at the unit place for subsequent operation. The second case arises when the sum of the negative digits (that is the digits with a bar) results in a number greater than $\bar{9}$. Once again we say Shudha, put a dot over the last bar digit just added, drop the unity at the tenth place, and retain the digit at the unit place for further operations. This digit is of course with a negative sign (and shall be written with a bar). Further, this to be noted that as each dot of Shudhikaran over a positive number is carried over as 1 in the next column (place), similarly each dot over a bar digit (which is essentially a negative digit) is carried over as unity with a bar to the next place. The following illustrations shall explain the process.

1·7·1 Illustration 12:

$$\begin{array}{r} 3\ \bar{2}\ \dot{5}\ 4 \\ +\ 2\ 5\ 6\ \bar{3} \\ \hline 1\ 1 \end{array}$$

Step 1 : At unit place $4+\bar{3}=1$

Step 2 : $5+6=11$ so we say 'Shudha' put a dot over

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5, and retain only 1 at the unit place.

Step 3 : Since there is a dot over a positive digit (5) at the previous place, we have 1 carryover to the next place i. e. hundredth place. Therefore positive digit 5 is increased by 1, $1+5=6$.

$6+\bar{2}=6-2=4$.

Step 4 : $2+3=5$

Therefore 5411 is the answer.

$$\begin{array}{r} 3\,\bar{2}\,\dot{5}\,4 \\ +\ 2\,5\,6\,\bar{3} \\ \hline 5\,4\,1\,1 \\ \hline \end{array}$$

1·7·2 Illustration 13 :

$$\begin{array}{r} 5\,\dot{\bar{4}}\,\dot{\bar{5}}\,\dot{\bar{4}}\,3\,2 \\ +\ 3\,\bar{5}\,\bar{4}\,\bar{6}\,2\,\bar{4} \\ \hline 7\,0\,\bar{0}\,\bar{0}\,5\,\bar{2} \\ \hline \end{array} \qquad =700048$$

Step 1 : $\bar{4}+2=\bar{2}$

Step 2 : $3+2=5$

Step 3 : $\bar{4}+\bar{6}=\overline{10}$ (Shudha); 0 retained

Step 4 : (a) $\bar{5}+\bar{1}=\bar{6}$

(b) $\bar{6}+\bar{4}=\overline{10}$ (Shudha); 0 is retained

Step 5 : $\bar{4}+\bar{1}=\bar{5}$,

$\bar{5}+\bar{5}=\overline{10}$ (Shudha); 0 is retained

Step 6 : $5+\bar{1}=4$,

$4+3=7$

Ans. $700005\bar{2}$, in normalize form 700048.

1·8 Subtraction of Vinculum Numbers (Part II)

While carrying on the subtraction of Vinculum numbers no borrowing may be required but instead a negative carry-over may be required. This shall be done by using

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the 'Sutra Shudha' as already explained in the previous section.

1·8·1 Illustration 14 :

$$\begin{array}{r} 3\,\dot{\bar{5}}\,2\,\dot{\bar{4}} \\ -\,2\,5\,1\,6 \\ \hline 0\,0\,0\,0 \\ \hline \end{array}$$

Step 1 : At unit place $\bar{4}-6=-10$ we say Shudha and carry over $\bar{1}$ to the next place and retain 0 at the unit place Put a dot over $\bar{4}$

Step 2 : Since—1 is to be carried over to the tenth place so either we add this 1 to the digit to be subtracted or reduce the upper digit by 1, $2-1=1$ therefore $1-1=0$ alternatively $1+1=2$ and $2-2=0$.

Step 3 : $\bar{5}-5=-10$.

We get a dot of Shudhikaran over $\bar{5}$ and retain 0.

Step 4 : $3-1=2$ and $2-2=0$, as in step 2 Hence we get 0000.

1·8·2 Illustration 15 :

$$\begin{array}{r} 3\,\dot{\bar{4}}\,\dot{\bar{5}}\,\bar{4}\,4 \\ -\,1\,5\,4\,6\,\bar{5} \\ \hline 1\,0\,0\,0\,9 \\ \hline \end{array}$$

$=10009$

Step 1 : At unit place $4-\bar{5}=4+5=9$.

Step 2 : $\bar{4}-(+6)=\bar{1}\bar{0}$ (Shudhikaran); retain 0.

Step 3 : (a) $-5+(-1)=-6$

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(b) $-6-(+4)=-10$ or $\bar{1}\bar{0}$ (Shudhikaran); retain 0

Step 4 : (a) $-4-1=-5$

(b) $-5-5=\bar{1}\bar{0}$ (Shudhikaran); retain 0,

Step 5 : (a) $3-1=2$

(b) $2-1=1$

$$\begin{array}{r} 3\,\bar{4}\,\bar{5}\,\bar{4}\,4 \\ -\;1\,5\,4\,6\,\bar{5} \\ \hline 1\,\bar{0}\,\bar{0}\,\bar{0}\,9 \\ \hline \end{array}$$

Since $\bar{0}=0$

So in normalized form $=10009$.

1 9 Discussions :

1. We observe interesting examples in the operation of Vinculum numbers We should develope a confidence in operation of Vinculum numbers.

2. There are a large number of applications of the Vinculum approach which simplify the arithmetical operations involved.

3. Further the Vedic Method of checking is available for spotting any random errors, which is outlined in the following chapter.

PROBLEMS :

Q. 1. Perform the addition of following Vinculum numbers and then simplify these :

(i) $$\begin{array}{r} \bar{1}\,\bar{2}\,\bar{3}\,\bar{4}\,5 \\ 3\,0\,\bar{1}\,8\,\bar{1} \\ \hline \end{array}$$

(ii) $$\begin{array}{r} 1\,3\,\bar{4}\,5\,\bar{1}\,2 \\ 3\,\bar{4}\,5\,\bar{1}\,2\,\bar{3} \\ \hline \end{array}$$

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(iii) $\bar{1}\ 3\ \bar{2}\ 2\ \bar{3}\ 1\ \bar{4}$
$4\ \bar{1}\ 3\ \bar{3}\ 4\ \bar{2}\ 1$

Q. 2. Perform the subtraction of following Vinculum numbers and then simplify the result with appropriate sign :

(i) $\begin{array}{r} 0\ 1\ 2\ 3\ 4\ 5\ 6 \\ -\ 6\ \bar{2}\ \bar{3}\ \bar{4}\ \bar{5}\ \bar{4}\ \bar{1} \\ \hline \end{array}$ (ii) $\begin{array}{r} 2\ \bar{3}\ 5\ 1\ 0\ 1 \\ -\ 4\ \bar{4}\ 1\ \bar{2}\ 1\ 0 \\ \hline \end{array}$

Q. 3. Perform following multiplication of Vinculum numbers :

(i) $7 \times \bar{2}$ (ii) $1\bar{1} \times 3$ (iii) $1\bar{2} \times \bar{4}$ (iv) $2\bar{3} \times 2$

Q. 4. Which of the following is true or false ? Correct the wrong solutions :

(i) $\begin{array}{r} 3\ \bar{7}\ 2 \\ +\ 1\ \bar{3}\ \bar{1} \\ \hline 5\ 0\ 1 \\ \hline \end{array}$ (ii) $\begin{array}{r} 3\ 5\ \bar{5} \\ +\ \bar{3}\ \bar{4}\ 4 \\ \hline 0\ 0\ 0 \\ \hline \end{array}$ (iii) $\begin{array}{r} 2\ 3\ 4 \\ +\ 1\ \bar{3}\ \bar{4} \\ \hline 1\ 0\ 0 \\ \hline \end{array}$

Q. 5. Perform the subtraction of following vinculum numbers and then simplify these :

(i) $\begin{array}{r} 3\ 0\ \bar{1}\ 8\ \bar{1} \\ -\ \bar{1}\ \bar{2}\ \bar{3}\ \bar{4}\ 5 \\ \hline \end{array}$ (ii) $\begin{array}{r} 5\ 4\ 5\ 3\ 4 \\ -\ 3\ \bar{5}\ \bar{5}\ \bar{4}\ \bar{7} \\ \hline \end{array}$

(iii) $\begin{array}{r} 6\ 5\ 4\ 3 \\ -\ 5\ \bar{4}\ \bar{6}\ \bar{3} \\ \hline \end{array}$

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CHAPTER 2

Mixed Addition and Subtraction

2 1 Introduction : In the last chapter we have learnt the easy method of addition and subtraction of Vinculum numbers. With a little practice we can become confident about the simple operations of Vinculum numbers. Very frequently we have to handle problems involving mixed addition and subtraction of several numbers The normal method would require a series of additions and subtractions. Alternatively, we can find the addition of all the positive as well as negative numbers separately. The final number is obtained by finding the difference of the two additions. But in the Vedic method it is possible to obtain the result in 'a single step This is done by converting all negative numbers (i. e. the numbers to be subtracted) to digits with bar and then adding the numbers, adopting the approach of addition of Vinculum numbers.

2·2 Illustration 1 : 374—253+245—433+327

Step 1 : We convert the negative numbers to the numbers with bar digits and then add.

$$
\begin{array}{r}
3\,7\,4 \\
\bar{2}\,\bar{5}\,\bar{3} \\
2\,4\,5 \\
\bar{4}\,\bar{3}\,\bar{3} \\
+\;3\,2\,\dot{7} \\
\hline
2\,6\,0 \\
\hline
\end{array}
$$

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Step 2 : We add the unit column $4+\bar{3}=1$; $1+5=6$; $6+\bar{3}=3$; $3+7=10$ we say shudha and put a dot over 7, and we retain 0 at the unit place.

Step 3 : Before going to the tenth place column we look for the dots of Shudhikaran in the lower place column In this case there may be dots over positive digits and/or negative (bar) digits. The same shall be carried over to the next column with a positive or negative sign respectively.

In this case we find only one dot in the unit place column over 7 therefore $+1$ is the carry over to the next column. $1+7=8$; $8+\bar{5}=3$; $3+4=7$; $7+\bar{3}=4$ $4+2=6$. It is interesting to observe that there are very few instances of carry over as the positive and bar digits cancel each other.

Step : 4 There is no carry over from the tenth place. $3+\bar{2}=1$; $1+2=3$; $3+\bar{4}=\bar{1}$; $\bar{1}+3=2$ therefore 2 is the answer at the hundredth place. So the final answer of this mixed addition and subtraction is 260.

Note : In case any of the numbers consists of larger digits we may first convert them to smaller digits by using the Vinculum approach this shall have a double advantage of firstly reducing the size of the digits to be handled and secondly reducing the number of dots of Shudhikaran.

2·3 *Illustration 2* : 275—334+499—289+534—327.

Step 1 : Here we may convert both 499 and 289 to the Vinculum form

$499=50\bar{1}$ and $289=3\bar{1}\bar{1}$ therefore $-289=\bar{3}11$ now we can add the numbers in the usual manner.

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Step : 2 We add the unit column $5+\bar{4}=1$; $1+\bar{1}=0$; $0+1=1$; $4+1=5$; $5+\bar{7}=\bar{2}$;

Step : 3 At tenth place $7+\bar{3}=4$; $4+0=4$; $4+1=5$; $5+3=8$; $8+\bar{2}=6$; so the answer at tenth place column is 6.

Step : 4 At the hundredth place column $2+\bar{3}=\bar{1}$; $\bar{1}+5=4$; $4+\bar{3}=1$; $5+1=6$; $6+\bar{3}=3$. So the answer is $36\bar{2}$. Therefore in normalized form=358.

$$
\begin{array}{r}
2\ 7\ 5 \\
\bar{3}\ \bar{3}\ \bar{4} \\
5\ 0\ \bar{1} \\
\bar{3}\ 1\ 1 \\
5\ 3\ 4 \\
+\bar{3}\ \bar{2}\ \bar{7} \\
\hline
3\ 6\ \bar{2} \\
\hline
\end{array}
$$

2·4 Illustration 3 :

2893—1477+1809—499—1434+999+4345+1434 +2343—1342.

Step 1 : Firstly we may convert 2893, 1809, —499 and 999 to the Vinculum form. Therefore $2893=3\bar{1}\bar{1}3$, $1809=2\bar{2}1\bar{1}$, $499=50\bar{1}$ or $-499=\bar{5}01$ and $999=100\bar{1}$. Thus we can simply add the numbers in usual manner.

Step 2 : Firstly we will add the unit column. $3+\bar{7}+\bar{1}+1+\bar{4}+\bar{1}+5+4+3+\bar{2}=1$

Step 3 : $\bar{1}+\bar{7}=\bar{8}$; $\bar{8}+1=\bar{7}$; $\bar{7}+0=\bar{7}$; $\bar{7}+\bar{3}=\bar{1}\bar{0}$ so we say Shudha and put a dot over $\bar{3}$ and retain 0 at tenth place. Further, we add $0+0=0$; $0+4=4$; $4+3=7$; $7+4=11$. Here again we say Shudha and put a dot over 4 and retain 1 at the tenth place further we add $1+\bar{4}=\bar{3}$.

Step 4 : In the tenth column there are two dots on $\bar{3}$ and 4 so $\bar{1}+1=0$ is the net carry-over. Hence we will add 0 to $\bar{1}$ which is

$$
\begin{array}{r}
3\ \bar{1}\ \bar{1}\ 3 \\
\bar{1}\ \bar{4}\ \bar{7}\ \bar{7} \\
2\ \bar{2}\ 1\ \bar{1} \\
\bar{5}\ 0\ 1 \\
\bar{1}\ \bar{4}\ \bar{3}\ \bar{4} \\
1\ 0\ 0\ \bar{1} \\
4\ 3\ 4\ 5 \\
1\ 4\ 3\ 4 \\
\dot{2}\ 3\ \dot{4}\ 3 \\
\bar{1}\ \bar{3}\ \bar{4}\ \bar{2} \\
\hline
1\ \bar{1}\ 1\ \bar{3}\ 1 \\
\hline
\end{array}
$$

Ans=9071

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$\bar{1}+0=\bar{1}$; $\bar{1}+\bar{4}=\bar{5}$; $\bar{5}+\bar{2}=\bar{7}$; $\bar{7}+\bar{5}=\bar{1}\bar{2}$, hence Shudha and put a dot over $\bar{5}$ and we retain $\bar{2}$ at the hundredth column and further we add $\bar{2}$ to $\bar{4}$ which is $\bar{2}+\bar{4}=\bar{6}$; $\bar{6}+0=\bar{6}$; $\bar{6}+3=\bar{3}$; $\bar{3}+\bar{4}=1$; $1+3=4$; $4+\bar{3}=1$.

Step 5 : There is a dot over $\bar{5}$ in the previous column so we add $\bar{1}$ to 3 which gives $3+\bar{1}=2$; $2+\bar{1}=1$; $1+2=3$; $3+4=7$; $7+1=8$; $8+2=10$ so we say Shudha and put a dot over 2, and retain 0 at the thousandth place and add $0+\bar{1}=\bar{1}$. But there is a dot over 2 so we write 1 before $\bar{1}$.

2·5 Illustration 4 :

234—176+583+378—289

$$
\begin{array}{rccc}
 & 2 & 3 & 4 \\
 & \bar{1} & \bar{7} & \bar{6} \\
 & 5 & 8 & 3 \\
+ & \dot{3} & \dot{7} & 8 \\
 & \bar{2} & \bar{8} & \bar{9} \\
\hline
1 & \bar{2} & \bar{7} & 0 \\
\hline
\end{array}
$$

$1\bar{2}\bar{7}0 = 730$ Ans.

2·6 Illustration 5 :

$$
\begin{array}{ccc}
4 & 3 & 4 \\
\bar{1} & \bar{5} & \bar{4} \\
\bar{1} & \bar{4} & \bar{5} \\
\bar{8} & 1 & 1 \\
\dot{\bar{4}} & \dot{\bar{4}} & \bar{5} \\
2 & \bar{4} & \bar{4} \\
\hline
\bar{1} & \bar{3} & \bar{4} & \bar{3} \\
\hline
\end{array}
$$

$= -1343$ A negative answer etc

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Problems :

Simplify given set of numbers as indicated below :

(i) 317 + 512—216 + 412 + 213—467 + 219

(ii) 3158 + 8196—816 + 8195 + 2163—956

(iii) 21893 + 8517—21385—81.8 + 26595—31493 —29617 + 8197

(iv) 6156—8953 + 41315 + 21689—5123—6895 + 81963—67314

(v) 319 + 21589—15317 + 2168—539 + 81963 —41957—213 + 156—181

(vi) 645—346 + 2196 + 3245 899—767 + 392

(vii) 987—1994—2386 + 4578—2488 + 999—88 (Using Nikhilam Sutra)

(viii) 2456 + 399 + 645—245 + 482–420—299

(ix) 999—888 + 777—666 + 555—444

(x) 425 + 524—646 + 864—22 + 111—989

(xi) 517 + 480—395—267 + 1199—0467

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CHAPTER 3

Checking Vinculum Operations

1 Introduction : We have studied simple and fast methods of checking the correctness of any addition and subtraction in Pushp 1. Further the easy method of checking random errors in any multiplication has been explained. The same step by step procedure can be applied for checking the correctness of any operation on the Vinculum numbers. There is very little modification required and the same shall be obvious from the following illustrations.

3·2 Addition Check : The steps required for checking the correctness of any addition of Vinculum numbers, are the same as those for simple addition (refer chapter 6 Pushp 1)

We convert each number to an equivalent single digit number and operate the single digit numbers. This should be equal to the single digit form of result.

3·2 1 Illustration 1 :

```
   3 2̄ 5̊ 4
  +2 5 6 3̄
  ------------
   5 4 1 1
  ------------
```

Step 1 : Add the digits of the original number to obtain the single digit number $3+(\bar{2})=1$, $1+5=6$, $6+4=10$, $1+0=1$.

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Step 2 : Add the digits of the number to be added $2+5=7$, $7+6=13$, $1+3=4$, $4+(\bar{3})=1$.

Step 3 : Add the two single digits obtained $1+1=2$.

Step 4 : Add the digits of the answer $5+4=9$, $9+1=10$; $1+0=1$, $1+1=2$.

Step 5 : Hence $2=2$, answer is correct. There are no random errors in the operation.

3·2·2 Illustration 2 :

$$\begin{array}{r} 4\,\bar{2}\,\dot{\bar{5}}\,3 \\ +\ 5\,3\,\bar{5}\,\bar{4} \\ \hline 9\,0\,0\,\bar{1} \\ \hline \end{array}$$

Step 1 : Add the first number to get the single digit number $4+(-2)=2$, $2+\bar{5}=\bar{3}$, $\bar{3}+3=0$.

Step 2 : Similarly $5+3=8$, $8+(-5)=3$, $3+(-4)=-1$ or $\bar{1}$.

Step 3 : Now add the two numbers obtained. $\bar{1}+0=.\bar{1}$

Step 4 : The result. $9+0=9$, $9+0=9$, $9+(-1)=8$.

Step 5 : NOTE : Here 9 is the turn point digit in checking system and not 10, because $10=1+0=1$. Therefore $\bar{1}=9-1=8$. Hence $8=8$ 0 kay.

3 2·3 Illustration 3 :

$$\begin{array}{r} 5\,\dot{\bar{4}}\,\dot{\bar{5}}\,\dot{\bar{4}}\,3\,2 \\ +\ 3\,\bar{5}\,\bar{4}\,\bar{6}\,2\,\bar{4} \\ \hline 7\,0\,0\,0\,5\,\bar{2} \\ \hline \end{array}$$

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Step 1 : $5+(\bar{4})=1,\ 1+\bar{5}=\bar{4},\ \bar{4}+\bar{4}=\bar{8},\ \bar{8}+3=\bar{5},\ \bar{5}+2=\bar{3}.$

Step 2 : $3+\bar{5}=\bar{2},\ \bar{4}+\bar{2}=\bar{6},\ \bar{6}+\bar{6}=\bar{1}\bar{2}$; (convert to single digit) $\bar{1}+\bar{2}=\bar{3};\ \bar{3}+2=\bar{1},\ \bar{1}+\bar{4}=\bar{5}.$

Step 3 : $\bar{3}+\bar{5}=\bar{8}$

Step 4 : $7+5=12;\ 1+2=3;\ 3+\bar{2}=1$

Step 5 : $9-8=1$ as 9 is the turn point (refer 3.2.2).

Step 6 : Hence $1=1$ Okay.

3·3 Subtraction : Like addition the steps for checking the subtraction of Vinculum numbers are similar to simple number subtraction. The additional points are brought out through the illustrative examples.

3·3·1 Illustration 4 :

$$\begin{array}{r} 4\ \bar{5}\ \dot{\bar{3}}\ 3\ \dot{\bar{7}} \\ -\ 2\ \bar{5}\ \bar{8}\ \bar{7}\ 4 \\ \hline 2\ 1\ 1\ 9\ \bar{1} \\ \hline \end{array}$$

Step 1 : $4+\bar{5}=\bar{1},\ 3+\bar{1}=2,\ 2+3=5,\ \bar{7}+5=\bar{2}$

Step 2 : $2+\bar{5}=\bar{3},\ \bar{8}+\bar{3}=\bar{1}\bar{1},\ \bar{1}+\bar{1}=\bar{2},\ \bar{2}+\bar{7}=\bar{9},\ \bar{9}+4=\bar{5}.$

Step 3 : $2+1=3,\ 3+1=4,\ 9+4=13;\ 1+3=4;\ 4-1=3.$

Step 4 : $\bar{5}+3=\bar{2}$ (add result of step 2 and step 3)

Step 5 : Hence $\bar{2}=\bar{2}$ Okay (same values at steps 1 & 4).

3·3 2 Illustration 5 :

$$\begin{array}{r} 5\ 6\ \bar{4}\ \dot{8}\ 3\ \bar{5} \\ -\ 4\ \bar{2}\ \bar{5}\ \bar{5}\ \bar{2}\ 4 \\ \hline 1\ 8\ 2\ 3\ 5\ \bar{9} \\ \hline \end{array}$$

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Step 1 : $5+6=11$; $1+1=2$; $\bar{4}+2=\bar{2}$, $8+\bar{2}=6$, $6+3=9$ $9+\bar{5}=4$

Step 2 : $4+\bar{2}=2$; $\bar{5}+2=\bar{3}$; $\bar{3}+\bar{5}=\bar{8}$; $\bar{8}+\bar{2}=\overline{10}$ $(=\bar{1})$; $4+\bar{1}=3$.

Step 3 : $1+8=9$; $9+2=11$; $1+1=2$; $2+3=5$, $5+5=10$; $1+0=1$, $1+\bar{9}=\bar{8}$.

Step 4 : $\bar{8}+3=\bar{5}$.

Step 5 : $9-5=4$ (As turn point is 9).

Step 6 : Hence $4=4$ Okay

3 4 Mixed Addition And Subtraction : This simple checking method can also be extended for the checking of the operations of mixed addition and subtraction. The steps involved are just similar to those used for checking the large addition sums of ordinary numbers (refer chapter 6 Pushp 1). The only additional point is that the turn point in checking is 9 (not 10 as in decimal system).

3 4 1 Illustration 6 :

Let us check the problem of illustration 2 of previous chapter.

	Single digit
2 7 5	5
$\bar{3}$ $\bar{3}$ $\bar{4}$	$\bar{1}$
5 0 $\bar{1}$	4
$\bar{5}$ 1 1	$\bar{1}$
5 3 4	3

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Let us check the solution

	Single digit
3 4 4	2
$\bar{1}$ $\bar{6}$ $\bar{7}$	$\bar{5}$
$\bar{2}$ $\bar{5}$ $\bar{4}$	$\bar{2}$
8 0 2	1
$\bar{2}$ $\bar{4}$ $\bar{7}$	$\bar{4}$
$\bar{1}$ $\bar{3}$ $\bar{6}$	$\bar{1}$
4 $\bar{5}$ $\bar{8}$ = $\bar{9}$	$\bar{9}$ Vertical addition c the single digits give

$\bar{9} = \bar{9}$ Hence Okay.

This simple check method can be applied in just a fe seconds to spot any random error.

3 5 Checking The Vinculum Numbers

It is interesting to note that this simple and strai forward Vedic Check Method for ascertaining the abse of any random error, can also be applied for checking corretness of the numbers after they are converted to Vinculum form and vice versa.

The simple procedure is just the same, with the ac ional facet of turning point at 9 and the correspon equivalence of the positive and the complimentary ne ive digits. The following examples shall ample illus the whole process.

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3·5·1 Illustration 8 :

The number 46313 in Vinculum form is $=5\bar{4}313$

Single digit equivalent $4+6=10$, $1+0=1$; $1+3=4$ $4+1=5$; $5+3=8$.

The Vinculum number similarly gives

$5+\bar{4}=1$; $1+3=4$; $4+1=5$; $5+3=8$.

We get the same single digit number (8) for both the normal and the Vinculum number. This is an indicator that there is no random error.

3·5·2 Illustration 9 :

The transformation of the vinculum number $2\bar{5}\bar{9}\bar{5}4$ into the normalised form gives 14054.

The vinculum number gives $2+\bar{5}=\bar{3}$; we ignore $\bar{9}$; $\bar{3}+\bar{5}=\bar{8}$; $\bar{8}+4=\bar{4}$.

The number gives $1+4=5$; $5+5=10$, $1+0=1$; $1+4=5$

But we know that $\bar{4}=9-4=5$. Hence, the check is satisfied.

Problems :

(1) Apply Vedic check method in respect of all the problems corresponding to Chapter 1 and 2.

(2) Write True/False for the following, use Vedic check method.

(i) $4597456=45135\bar{4}\bar{4}$

(ii) $98976423=0\bar{1}\bar{1}\bar{3}\bar{4}423$

(iii)
$$\begin{array}{r} 3\ 4\ \bar{5}\ \bar{6} \\ +\ \bar{2}\ 5\ \bar{4}\ \bar{3} \\ \hline 5\ 9\ \bar{8}\ \bar{9} \\ \hline \end{array}$$

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(iv)

$$\begin{array}{r} 5\,4\,6\,7 \\ -\,2\,\bar{4}\,\bar{6}\,\bar{8} \\ \hline 3\,0\,0\,5 \\ \hline \end{array}$$

(v)

$$\begin{array}{r} 5\,4\,\bar{3}\,7 \\ 2\,\bar{4}\,5\,6 \\ 3\,\bar{5}\,\bar{5}\,6 \\ \hline 1\,0\,\bar{5}\,\bar{5}\,6 \\ \hline \end{array}$$

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CHAPTER 4

Compound Addition an Subtraction

4·1 Compound Subtraction : The process of co subtraction in Vedic Method is converted to a s simple subtraction and addition. Once again th 'Shudha' is deployed whenever the larger numb particular unit is to be subtracted from the number of the same unit. It is to be noted that method there is absolutely no need to convert units to a single unit as is frequently done in th procedure. The step by step process is illustrate following examples :

4.1.1 Example :

	5 yds	1 ft	9 inches
1	— 2	2	11

Step 1 :

The number 11 at inches place is bigg corresponding number 9. Therefore Shu required.

(a) We say Shudha and put a dot abov number at the next unit (2).

(b) Find the compliment of the lower num unit size (12) in this case as there are +12 in

the is wer the 1 ft.

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number 11 (from 1 ft the next unit in this case).

(c) Add this compliment to the upper number. 1+9 is the answer.

5 yd	1 ft	9 inch
— 2	·2	11
		10

Step 2 :

We move on to next unit (which is 'ft' in this Since the lower number has a dot above this, we in it by 1. Therefore 2+1=3 and 3 is larger tha corresponding number at this place (which is 1 i case). 'Sudhikaran' is required.

(a) We say 'Sudha' and put a dot above the number at the next unit (2 yds here.)

(b) The compliment of this lower number is co by subtracting it from the unit size, which is 3 case (as there are 3 ft in 1 yard). Therefore 3— the compliment of the increased lower number 3.

(c) Add this compliment to the upper number. fore 1+0=1 is the answer.

5 yd	1 ft	9 inches
— 2	2	11
	1	10

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Step 3 :

We move on to the next unit which is yards in this case. Since the lower number has a dot above this, we increase it by 1.

Therefore $2+1\dot{=}3$. In this case the lower number (3) is smaller than the upper number (5) as such $5-3=2$ yards is the answer. Therefore the final answer is 2 yards 1 ft and 10 inches.

Note : We observe that in the Vedic Method of compound subtraction there is no carry over of the unit instead we only find out the compliment of each number. The later being easier, hastens the computation process and also reduces the chances of errors.

4·1·2 Illustration 2 :

1 Decade	3 yrs	3 months	5 days	13 hrs	50 min	30 seconds
0	2	4	5	15	55	25

Step 1 :

$30-25=5$ secs is the straight answer for the first unit (secs in this case).

Step 2 :

55 min is larger than the corresponding upper number (50). Therefore 'Shudhikaran' is carried on. We put a dot above the number at the next unit (15 hrs). The compliment of 55 is 5 (60—55), which is added to the upper number (50), therefore $5+50=55$.

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1 decade	3 yrs	3 months	5 days	13 hrs	50 min	30 seconds
0	2	4	5	1$\dot{5}$	55	25
					55	5

Similarly the process is repeated to complete the computation.

1 decade	3 yrs	3 months	5 days	13 hrs	50 min	30 seconds
0	$\dot{2}$	$\dot{4}$	$\dot{5}$	1$\dot{5}$	55	25
1	0	10	29	21	55	5

4.2 Compound Addition :

The process of compound addition in Vedic method is very similar to the compound subtraction. The sutra invoked is also the same, namely 'shudha' (शुद्धा:). The process of 'Shudhikaran' is done whenever the addition of numbers at a particular unit exceeds the unit size. The steps involved are similar to those in subtraction. The basic differences are (i) the compliment is computed for the larger number (ii) the compliment is subtracted from the smaller number to get the answer.

4·2·1 Illustration 3 :

4 yards	1 ft	11 inches
3	2 ft	8 inches

Step 1 : The larger number is 11 inches. The process of 'shudhikaran' is carried out, as it is very close to the unit size (12 inches) in this case.

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(b) Compute the compliment
(12−11=1).

(c) Subtract this compliment from the smaller num
8−1=7 is the answer.

4 yards	1 ft	11 inches
3 yards	2 ft	8 inches
		7

Step II : We move on to the next unit (here ft). the larger number has a dot above this, we increase 1, 2+1=3 ft. The process of 'Shudhikaran' is carrie

(a) We say 'Shudha' and put a dot above the number at the next unit place.

(b) Compute the compliment of the larger n 3−3=0, the compliment when subtracted fro smaller number (1 ft) gives the answer. 1−0=

4 yards	1 ft	11 inches
3	2	8
	1	7

Step III : We move to the next unit, which is this case. The larger number (4) has a dot a Therefore, it is increased by 1 (4+1=5) and adde smaller number, 5+3=8 yards is the answ process of 'Shudhikaran' is not required, as this u very large unit size.

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4.3 Vedic Check : In chapter 3, we have learnt rather simple and fast Vedic methods for checking the correctness of any subtraction and addition. This is a unique feature of the Vedic mathematics. At this stage the question naturally arises whether the unique Vedic sutra invoked for the checking of different operations can also be deployed even for compound addition and subtraction. The obvious answer is a positive 'YES'. The same process of converting to the equivalent single digit is used only with vary little modification The step by step processes are outlined in the following sections :

4 3 1 Checking of Compound Subtraction :

First of all let us consider the case of compound subtraction. In addition to the steps already considered for checking the vinculum subtraction we have to take care of the unit size and also the dot due to 'Shudhikaran' at each unit place. Further the checking can be conveniently carried on for each unit separately.

4 3.2 Example 4 :

Let us understand the step by step process with the help of following example :

5 yards	1 ft.	2 inches
— 2	2	11
2	1	10

The compound subtraction can be carried on by using the 'shudhikaran' process. We observe that the dot of Shudhikaran has appeared in the feet as well as yards

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larger numbers being subtracted at

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the inch and feet places, respectively. The check is applied separately for each unit.

STAGE I : Let us first take the inch unit.

Step 1 : The result 10 is converted to a single-digit number, $1+0=1$.

Step 2 : The number subtracted (11) is converted to a single digit number, $1+1=2$. Further if we observe the dot of 'Shudhikaran' appearing on this number we increase it by 1.

Step 3 : Now we add the single digit of the result to the equivalent single digit of the number subtracted as obtained in step 1 and 2.

Step 4 : Similarly the original number is also converted to a single digit number. Here it is just 9. We observe the number at higher unit. If it has a dot of 'shudikaran', we shall also add the unit size (12) to the original number and of course we shall reduce it to the single digit form. In the present case, we find the 'shudikaran' dot over the number 2 in the next unit 'feet'. Therefore, we add the unit size (1 feet=12 inches) to the original number. Therefore $9+12=21$, $2+1=3$. In the present example we get 3.

Step 5 : The Check—If our compound subtraction has been correctly done, we should get the same single—digit numbers in steps 3 and 4 respectively. In the present example we have obtained 3. This is a reasonable proof of the correctness of the operation at the inch unit.

STAGE II : After checking the computations at inch unit we repeat the same steps for the next feet unit.

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Step 1 : The result is just 1.

Step 2 : The number subtracted is just 2. But since we observe a 'shudhikaran' dot above 2, we increase it by 1. Therefore 2+1=3.

Step 3 : Now we add the single digit of the result to the equivalent single digit of the number subtracted as obtained in steps 1 and 2. In the present case 3+1=4.

Step 4 : Once again the original number is just 1. Further we observe that there is a 'shudhikaran' dot above the number subtracted at the next unit, namely, yards, Therefore we add the unit size (1 yard=3 feet) to the original number. In the present case we get 1+3=4.

Step 5 : The Check—We compare the numbers obtained in steps 3 and 4 to ascertain the correctness. Here we have obtained the same number 4.

STAGE III : Now we come to the unit-yards

Step 1 : The result is just 2.

Step 2 : The subtracted number is also 2. But since it has a 'shudhikaran dot' we increase it by 1, and get 2+1=3.

Step 3 : The addition of the single digits of the result and the number subtracted gives 2+3=5.

Step 4 : The original number is just 5.

Step 5 : On comparing the single digit numbers in steps 3 and 4 we find that both are 5 each. Hence the computations are correct.

4.3.3 Discussions :

(i) We have just learnt the rather simple method of checking the compound subtraction.

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(ii) Though the explanation has occupied a large space for clearity of presentation and understanding of the process, the actual operations can be performed just mentally.

(iii) The checking process would require just a few seconds time.

(iv) The same process is further applicable with obvious modification for checking the compound addition also (Refer Chapter 3).

(v) It is interesting to point out that in the present tedious method of compound subtraction, the only way for checking is to add the result with the number subtracted which would require rather large amount of effort. Alternatively, one may resort to the frequently adopted method of repeating the operation which has the obvious risk of doing the same mistake over again.

PROBLEMS :

Q. 1. Perform the following compound additions, using the Vedic 'Sudhikaran' Process, which is quicker, simpler and hence has lesser chances of errors.

(a)
2 hrs	31 min	29 sec
+ 0	59	42

(b)
8 yd	0 ft	9 inch
+ 6	2	11

(c)
33 tone	11 kgs	9 gm
+ 21	15	18

(d)
10 km	245 meter	89 cm—
+ 3	900	91

(e)
2 year	7 months	26 days	14 hrs.	11 minutes
+ 0	9	12	23	56

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CHAPTER 5

Compound Multiplication-Square and Cubic Measures

5,1 Introduction : By now we have seen vast applications of the simple Vedic sutras, in addition subtraction, tables and multiplication operations in pushp 1. We have always found the Vedic methods to be much faster than the normal methods. Very frequently we have observed the Vedic sutras to be working at almost a magic speed. Although it is not really a magic, as it is a complete system with sound base, which can be learnt, practised and then used by any aspirant. Specifically, the Vedic method for compound subtraction and addition is decidedly far more easier and faster. Now we come to the compound multiplication.

5 2 Computing Squares

The present method provides two alternative approaches for solving problems of compound multiplication. For example, let us take the problem of finding the area of a rectangular piece of land having dimensions in yard and feet (say the plot is of size 14 yard 2 feet x 23 yard 2 feet).

5 2·1 The Conventional Methods

The first method is to convert both of these dimensions of the plot into the smaller unit of dimensions (here feet). Then we multiply these whole number dimensions to get

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the area in squre units. Now we are required to divide this by square of the unit size to get the required area in two units.

Exampe : In the present case, the dimensions are 14 yards 2 feet by 23 yards 2 feet.

Therefore 14 yards 2 feet $=(14\times3)+2$
$=42+2=44$ feet

and 23 yards 2 feet $=(23\times3)+2$
$=69+2=71$ feet

The area is

```
    7 1
  × 4 4
  -------
    2 8 4
  2 8 4 ×
  -------
  3 1 2 4 sq. ft.
```

Now we have to convert this back into sq. yards and sq ft. Therefore.

```
  ----------
9 ) 3 1 2 4 ( 3 4 7
    2 7
    -----
      4 2
      3 6
      -----
        6 4
        6 3
        -----
          1
        -----
```

Therefore, the area is 347 sq. yards and 1 sq ft. The

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whole process is unnecessarily long, tedious and time consuming Now let us try to do the same problem by using the second conventional alternative method The first step in this case is to convert both the given dimensions into always rather vulgar fractions of the higher unit (in this case yards). This is followed by the multiplication of these fractions and then the rather tedious process of converting these fractions back to units.

Therefore. 14 yards 2 feet

$$=\frac{14\times 3}{3}+\frac{2}{3}=\frac{42}{3}+\frac{2}{3}=\frac{44}{3}$$

and 23 yards 2 feet $=\frac{23\times 3}{3}+\frac{2}{3}=\frac{69}{3}+\frac{2}{3}=\frac{71}{3}$

Now we multiply these numbers.

$$\frac{44}{3}\times\frac{71}{3}=\frac{3124}{9}$$

Further, we have to convert this back into sq. yards and sq feet

Therefore,

```
9 ) 3 1 2 4 ( 3 4 7 sq. yards
    2 7
    -----
      4 2
      3 6
      -----
        6 4
        6 3
        -----
          1
        × 9
        -----
9 ) 9 ( 1 sq. ft.
    9
    ---
    ×
```


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Therefore, area of the plot=347 sq yards and 1 sq. ft.

5·2·2 The Vedic Method

Both of these alternatives are too long, tedious, time-consuming and prone to error. However. the Vedic approach is rather simple and straight forward. Instead of resorting to the convertion to a single unit, we use the algebraic multiplication concept and diploy the 'Adyam sutra'. It can also be done directly without the use of algebra by Urdhva Sutra **(Ref. Item 5.6)**

Let us say that 'A' is the unit size, In the present example 1 yard=3 ft. Therefore A=3

Side 1=14 A+2 and Side 2=23A+2

We know area=(Side 1) × (Side 2)

Therefore, area=(14A+2) × (23A+2)

$$=322A^2 + 74A + 4$$

Now the middle term is split into two parts When we divide the coefficient of A by the unit size, Part 1 is the whole number and part 2 is the remainder.

Here the unit size is 3. Therefore, 74=72+2= 3×24+2. The quotient=24 and remainder=2.

Therefore area=$322A^2 + (24\times3+2)\,A+4$

$=322A^2 + (24\times3+2)\,A+4$ (As A=3) (i)

$=322A^2 + (24A+2)\,A+4$ (ii)

$=322A^2 + 24A^2 + 2A+4$ (iii)

$=346A^2 + 2\times3+4$ (iv)

$=346A^2 + 10=346A^2 + 9+1$ (v)

$=346A^2 + 3\times3+1$ (As A=3) (vi)

$=346A^2 + A^2 + 1$ (vii)

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$=347A^2 + 1$ (viii)

$=347$ sq. yards $+ 1$ sq. ft. (ix)

Since A^2 is the squared unit size $(3 \times 3 = 9)$. So dropping A^2 essentially means, the coefficient is automatically expressed in the squared higher unit (here sq yard). Further, the terms with A are transformed to the lower unit (here ft.) by substituting the value of A (step iv). The constant term (10) obtained at (v) step is larger than the squared unit size, we can split the same into two parts.

5·3 Illustration 2

Now let us take another example. Let we take the problem of finding the area of a rectangular plot which has the dimensions as 7 feet 8 inches by 5 feet 11 inches.

Let us say that A is the unit size in the present example ($A = 12$; as 1 ft. $= 12$ inches). We know that the area of the plot $=$ Side $1 \times$ Side 2.

Therefore, Area $= (7A + 8) \times (5A + 11)$

$= 35A^2 + 117A + 88$

$= 35A^2 + (9 \times 12 + 9)\ A + 88$

$= 35A^2 + (9A + 9)\ A + 88$ (as $A = 12$)

$= 35A^2 + 9A^2 + 9A + 88$

$= 44A^2 + 9 \times 12 + 88$

$= 44A^2 + 108 + 88$

$= 44A^2 + 196$

$= 44A^2 + 144 + 52$

$= 44A^2 + (12 \times 12) + 52$

$= 44A^2 + A^2 + 52$

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$=45A^2+52$

$=45$ sq. ft $+52$ sq inches

5·4 Example 3

In this example, we will try to find the area of a grassy rectangular field having dimensions as 3 metre 22 cm by 5 metre 10 cm. Here A is the unit size ($A=100$, as $1m=100$ cm.)

Therefore, Area $=(3A+22)\times(5A+10)$

$=15A^2+140A+220$

$=15A^2+A^2+40\times100+220$

$=16A^2+4220$

$=16.$ sq. m. $+4220$ sq. cm

Here 4220 is not divided into two parts as it is smaller than the squared unit size (10,000). Till now we have confined the Vedic method for finding the area of only rectangular figures. Further we can take the examples of finding the area of the parallelogram and trapezium. The same 'Adyam' sutra is applicable in these cases too. In the case of finding the area of trapezium, we first have to find the average of the two parallel sides and then multiply the average with the vertical distance between these two paraller sides.

Further, we know that the area of a parallelogram is equal to the product of the length of any one of the two sets of the parallel sides and the vertical distance between them.

5·5 Area of a trapezium

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Now we can take the example of finding the area of a trapezium with dimensions of the two parallel sides as 3 ft 10 inches (say x) by 7 ft. 8 inches (say y) and the distance between these two parallel sides is 2 ft 5 inches (say h).

From simple geometry, we know that the area of a trapezium with dimensions as given $=\frac{h}{2}(x+y)$ Once again if A is the unit size (here A=12).

$$\text{Area}=\frac{2A+5}{2}\Big((3A+10)+(7A+8)\Big)$$
$$=\tfrac{1}{2}(2A+5)\times(10A\times18)$$
$$=(2A+5)\times\left(\frac{10A+18}{2}\right)$$
$$=(2A+5)\times(5A+9)$$
$$=10A^2+43A+45$$
$$=10A^2+3A^2+7A+45$$
$$=13A^2+129$$
$$=13 \text{ sq. feet}+129 \text{ sq. inches}$$

In the present case too, the constant term 129 is not divided into two parts because it is smaller than the squared unit size and is written as such in sq. inche unit. Therefore, the area of the trapezium=13 sq. feet and 129 sq. inche.

5 6 Direct Compound Multiplication

In the previous sections we have seen the use of algebraic approach for compound multiplication. In the Vedic method, we can also do the compound multiplication

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directly by using the 'Urdhva Sutrs.

(Refer Puahp 1 x' chapter 8)

5 6.1 Illustration 5 :

Let us say we want to find the area of a plot of size 4 yards 1 feet by 8 yards 2 feet.

4	yd	1	feet	
8	yd	2	feet	
32	/	16	/	2

Step 1 : We use the Urdhva Sutra to obtain three parts the answer.

(1) R. H. S.=vertical product of RHS column

$=2\times1=2$

(2) Middle part=cross multiplication.

$=1\times8+2\times4=16$

This is entered in the middle portion.

(3) L. H. S=vertical product of LHS column

$=4\times8=32$

Step 2 : Now we split the middle portion into two parts. First part consists of that number which when divided by the unit size gives the maximum whole number quotient (here 1 yard=3 feet, therefore 3 is the unit size). And the second part is the remainder which is always less than the unit size.

In the present example $5\times3=15$ is the first part (which gives the maximum whole number quotient=5) and $16-15=1$ is the second part

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32 / 16 / 2
32 / 5×3+1/2

Step 3 : In the first part we drop the unit size (here 3) and carry the multipler (here 5) to LHS and add it to the value already there.

Here, we carry 5, therefore 32+5=37 square yards.

The second part (which is the remainder) is multiplied by unit size and the product carried to RHS. This is added to the value already there.

Here 1×3=3 is carried to RHS and added to the previous value=2, which is already there. Therefore 2+3=5 square feet is the answer.

32 / 5×3+1 / 2
= 32+5 / / 1×3+2
= 37 square yards/5 square feet

This example explains the simple Vedic method of compound multiplication without the use of algebra.

5 6 2 Illustration 6 :

Let us take another example. 8 feet 11 inches by 11 feet 7 inches.

8	feet	11	inches	
11	feet	7	inches	
88	/	177	/	77

Step 1 : We invoke the Urdhva Sutra

(1) RHS=11×7=77

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(2) Middle part=cross multiplication

$$=11\times11+8\times7=121+56=177$$

(3) LHS$=8\times11=88$

The values are entered as usual.

Step 2 : Here unit size is 12 (1 feet=12 inches). Therefore, we split the middle portion as $177=12\times14+9$.

Step 3 : We drop unit size 12, carry the multiplier 14 to LHS and add it to the value already there. Therefore $88+14=102$. Furher the second part (9) is multiplied with the unit size, carried to RHS and added to the previous value. Here we get $9\times12+77=108+77=185$

$$88/177/77=88/12\times14+9/77$$
$$=88+14/\quad/9\times12+77$$
$$=102/\quad/185$$

Further, we observe that the result in the lower unit (inch) is more than the squared unit size. Therefore, we carry the excess to squared feet

Therefore $185=1\times144+41$. We carry 1 to LHS.

The result $102/185=102/1\times144+41=102+1/41=$ 103 square feet and 41 square inches. This may also be written as 11 square yards+4 square feet and 41 square inches.

5 7 Vinculum Approach :

In compound multiplication, the computation work can be further simplified by first converting the larger of the smaller unit values into the vinculum form and then

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simplifying.

5·7·1 Illustration 7 :

8 feet 10 inches by 10 feet 3 inches. Here we first of all convert the larger inch dimension to vinculum form by taking its compliment from the unit size, which is written with a bar, and increase the higher unit by 1.

Here 8 feet 10 inches, the compliment of 10 inches= 12−10=2 which is written with a bar. Further the higher unit (8 feet) is increased by 1. Therefore 8+1=9 feet.

9	feet	$\bar{2}$
1 0		3
9 0 /	7 /	$\bar{6}$

Step 1 : Using Urdhve Sutra we get the three parts of the answer.

(1) RHS=$\bar{2}\times3=\bar{6}$

(2) $\bar{2}\times10+9\times3=-20+27=7$, is the middle protion.

(3) $9\times10=90$ is the LHS.

Step. 2 : The middle portion is less than the unit size (12). Therefore, nothing is to be carried to LHS. We multiply it by the unit size and carry it to RHS. Therefore, $7\times12=84+\bar{6}=78$ square inch.

Hence, we get 90 / 7 / $\bar{6}$=90/ /$7\times12+\bar{6}$
=90/78=90 square feet and 78 square inches

This can also be written as 10 square yards and 78 square inch.

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5·8 Computing Cubes :

Let us try to find the volume of a parallelopiped whose dimension are 3 ft 7 inches, 5 ft 10 inches and 7 ft 2 inches.

Step. 1 : The unit size is equal to 12. We first of all compute the area in the usual vedic manner.

$$\text{Area} = (3A+7)\times(5A+10)$$
$$= 15A^2+65A+70$$

Step 2 : For volume, we further multiply this by the third dimension and then transform the middle terms (have powers of A 1 and 2) to the upper power and the remainder portion to the, constant term by the usual vedic split process, using the 'Adyam' sutra.

$$\text{Volume} = 15A^2+65A+70$$
$$7A + 2$$
$$\overline{105A^3+30A^2+455A^2+490A+130A+140}$$

$$= 105A^3+485A^2+620A+140$$
$$= 105A^3+12\times40A^2+5A^2+12\times51A+8A+140$$
$$= 105A^3+40A^3+51A^2+5A^2+8\times12+140$$
$$= 145A^3+56A^2+96+140$$
$$= 145A^3+12\times4A^2+8A^2+236$$
$$= 149A^3+1152+236$$
$$= 149A^3+1388$$
$$= 149 \text{ cu. ft.} + 1388 \text{ cu. inch}$$

5 9 Vedic Check Method :

In the previous sections we have just seen how the problems of compound multiplication can be performed

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very easily by using the Vedic method which is both easy and superfast It completely eliminates the requirement of conversion to single unit right in the beginning of this type of problem At this stage, the question naturally-arises that is there a Vedic method for checking the correctness of the compound multiplication result. The answer once again is a positive 'YES' The present system has no way of checking this class of problems.

It is interesting to note that the same step by step Vedic check method which is used for multiplication is applicable for compound multiplication only with a slight modification Once again the same sutra 'Samucchya is at work The simple step by step process is outlined with the help of following examples.

5.9. Example 9 :

Let us take the example of multiplying 14 yards 2 ft by 23 yards 2 ft.

$$\begin{array}{l} 14A + 2 \\ 23A + 2 \\ \hline 322A^2 + 74A + 4 \\ \hline = 347A^2 + 1 \end{array}$$

Step 1 : Write down the multiplicand, multiplier and the result in three rows and draw a big square in front of it. Join the two diagonals, thereby dividing the inner space into four triangles.

Step 2 :

(i) Add the digits of the coefficient of the higher unit (if this consists of more than one digit).

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(ii) The resulting single-digit number is multiplied by the unit size and the resulting number is reduced to a single digit number.

(iii) The resulting single-digit number is then added to the number at the lower unit place However, if this yields a two-digit number, we once again convert it into a single digit by adding the digits in the usual manner.

(iv) The final single-digit number is written in the upper triangle.

In the present example,

(i) 14 gives $1+4=5$

(ii) Unit size is 3. Therefore, $5\times3=15$; $1+5=6$.

(iii) 6 is added to 2 feet; $6+2=8$.

(iv) The resulting single-digit number is written in the upper triangle

14 yards 2 feet

23 yards 2 feet

---

347 yards 1 sq ft.

Step 3 :

We add the digits of the lower compound number by adopting exactly the same process The final resulting single digit number is written in the lower triangle.

In the present example,

(1) 23 gives $2+3=5$

(ii) Unit size is 3. Therefore, $5\times3=15$; $1+5=6$.

(iii) 6 is added to 2 feet; $6+2=8$.

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(iv) The resulting single-digit number is written in the lower triangle.

Step 4 :

Now as in the multiplication check we multiply the single-digit number in the upper triangle with the number in the bottom triangle. We write the result adjacent to the right hand side triangle. Add the digits of the resulting number to get a single-digit number which is then entered in the R. H. S. triangle.

In the present example, we get $8 \times 8 = 64$; $6 + 4 = 10$; $1 + 0 = 1$.

Step 5 :

Add the digits of the higher unit of the result. This is multiplied with the square of the unit size and the result is converted to a single-digit number The number thus obtained is finally added to the number at the lower unit, and the result is once again converted to a single-digit number in the usual manner. In the present example,

347 gives $3 + 4 + 7 = 14$; $1 + 4 = 5$ This is multiplied with the square of the unit size (1 yard = 3 feet). Therefore, $5 \times 9 = 45$; $4 + 5 = 9$ Finally $9 + 1 = 10$; $1 + 0 = 1$. This 1 is entered in the L. H. S triangle.

Step 6 :

The Check—If the number in L H S triangle is equal to the number in R.H.S. triangle, we can be reasonably sure about the correctness of the answer.

In the present example, we have obtained 1 in both these triangles Okay.

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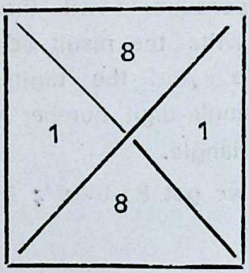
14 yards	2 feet
23 yards	2 feet
347 sq. yards	1 sq. ft.



$8 \times 8 = 64;$
$6 + 4 = 10;$
$1 + 0 = 1;$

5·10 Discussions :

1. The present approach of compound multiplication calls for unnecessary single-unit conversions which is followed by lot of operations with the usually terrifying vulgar fractions and the whole process is inflicting the student's mind.

2. But the Vedic method on the other hand makes the whole process rather simple and easy and almost completely eliminates handling of the vulgar fractions.

3. This approach also reduces the chances of error.

4. The step by step Vedic check method once properly understood can be straight away applied through mental arithmetic and only the final single digit number of each step needs to be written down in a corner. Once the method was been properly understood and practised, we may not require even the use of the square.

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5. It is important to point out that in checking the problems of square measures the error in the higher unit shall elude the check where the summation of the squared unit size gives 9 due to obvious reasons.

PROBLEMS :

1. Compute the following :—

   (i) 3' 8" × 2' 11" (ii) 1 yd 2' × 5 yd 1'

   (iii) 2 yd 2' 3" × 3 yd 1' 7"

   Apply Vedic check method in all above cases.

2 Compute volume in case of following parallelopiepeds whose dimensions are—

   (i) 2' 5", 2' 11", 6' 8"

   (ii) 4' 10", 3' 9", 2' 8"

   Answer should be in term of cubic ft and cubic inches only. Apply the check.

3. Compute the area of trapizium whose lengths of parallel sides and their separation is as follows :—

	Lengths (1)	Lengths (2)	Separation
(i)	3' 6"	2' 7"	1' 3"
(ii)	5' 8"	3' 11"	2' 6"

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CHAPTER 6

Nikhilam Method-Using Sub base

6·1 Introduction :—In Pushp-1 (chapter 7) we have learnt the use of NIKHILAM method for multiplication of numbers closer to the base.

However, frequently we have to multiply two numbers neither of which is near a convenient base. The solution of these problems can be obtained by using a small 'Upsutra'; 'Anurupyena' (अनुरूपेण) which means proportionately, when applied, it means in all cases where there is a rational relationship (ratio), the ratio should be taken into account. This ratio shall lead to a proportionate multiplication or division as the case may be.

(Pushp—1 Chapter—8)

6·2 Illustration 1 :

Multiply 48×48

Case I Base $=100$

Working base (W.B.) $=50$

Submultiple $=\frac{1}{2}$ (because $100 \times \frac{1}{2} = 50$ (WB)

48 — 02
48 — 02

46 / 04

L.H.S. = Submultiple × 46
$= \frac{1}{2} \times 46 = 23$

Therefore the result is 23/04 = 2304

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Note : Since the base is 100, therefore two digits are essential on the R.H.S.

Case II Base $=10$
Working base $=50$
Multiple $= 5$ (because $5 \times 10 = 50$ (WB)

$$\begin{array}{r} 48 - 2 \\ 48 - 2 \\ \hline 46 \;/\; 4 \end{array}$$

L.H.S. $=$ Multiple $\times$ 46
$= 5 \times 46 = 230$

Therefore the result is $= 230/4 = 2304$

Note : Since the base is 10, therefore, only one digit is sufficient on the R.H.S.

Case III Base $=10$
Working base $=40$
Multiple $= 4$ (because $4 \times 10 = 40$ (WB)

$$\begin{array}{r} 48 + 8 \\ 48 + 8 \\ \hline 56 \;/\; 64 \end{array}$$

Since the base is 10, only one digit is required on the on the R.H.S. Therefore 6 is to be carried over to the L.H.S. However, it is important to note that the carry over from the R.H.S. will be added only to the final L.H.S. answer. The L.H.S. is obtained after applying the sutra 'Anurupyena'.

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L.H.S. = Multiple × 56
= 4 × 56 = 224

We get

48 + 8
48 + 8

224 / 64 = 230/4
= 2304

Comments : (i) Each case of this illustration clearly demonstrates the use of the Sutra 'Anurupyena'.

(ii) As already observed we get the same answer by adopting any one of the above mentioned alternative methods. This is one of the main features of the Vedic Mathematics. The new student can solve the problem by one method and check the solution by resolving by other methods.

6·3 Illustration 2 :

Multiply 252 × 254

Base = 1000

Working base = 250

Multiple = $\frac{1}{4}$ (because $1000 \times \frac{1}{4} = 250$

252 + 002
254 + 004

256 / 008
×
1/4

= 64/008 = 64003

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6 4 Illustration 3 :

Multiply 42 × 43

Case I Base =100
Working base =50
Submultiple $=\frac{1}{2}$

```
42 — 08
43 — 07
--------
35 / 56
```

The L H.S. = Submultiple × 35
$= \frac{1}{2} \times 35 = 17\frac{1}{2}$

Since 35 is an odd number it's division by 2 gives $\frac{1}{2}$ on the L H S. Just as $\frac{1}{2}$ Rupees means 50 paise the $\frac{1}{2}$ here means base $\times\frac{1}{2}=100\times\frac{1}{2}=50$, which should be taken over to the R.H S. (as 50) and added to the already obtained R.H.S.

```
42 — 08
43 — 07
--------
35 / 56
 ×
1 / 2
```

```
= 42 — 08
  43 — 07
  --------
  17 / 56 + =17/106
       50
```

Therefore the R.H.S. is obtained as 56+50=106.

But since the base is 100 only two digits are permissible on R.H.S. Therefore, the excess 1 is carried to L.H.S. as usual.

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42 — 08
43 — 07

17 / 06 = 17+1/06=18/06
1 =1806

Case II Base =10
Working base=50
Multiple = 5

42 — 8
43 — 7

35 /6
5×/5

= 42 — 8
43 — 7

175 / 6 = 1806
5

Remark :

(i) Case II yields the answer very quickly. Therefore, it is very important to select a suitable combination of base and working base.

(ii) It will be useful to observe that if the difference of the two numbers is odd, taking a combination of base and sub-base, which requires a sub-multiple, would lead to longer calculations. However, a combination requiring multiple would give results quickly.

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Case III Base $=10$
Working base $=40$
Multiple $= 4$

$$\begin{array}{r} 42 + 2 \\ 43 + 3 \\ \hline 45 \;/\; 6 \\ \times 4 \end{array}$$

$= 180/6 = 1806$

Remarks : The computational efforts are further reduced by taking a working base closer to the numbers (in this case 40). In this case no carry over is required.

65 Illustration 4 : 506×484

Base $=1000$
WB $= 500$
Submultiple $= \frac{1}{2}$

$$\begin{array}{r} 506 + 006 \\ 484 - 016 \\ \hline 490 \;/\; \overline{096} \\ \times \frac{1}{2} \;/ \end{array}$$

$=245 \;/\; \overline{096}$
$=244 \;/\; 1000\text{-}96$
$=244 \;/\; 904$
$=244904$

66 Illustration 5 : 389×512

Base $=1000$
WB $= 500$
Submultiple $= \frac{1}{2}$

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$$\begin{array}{r} 389 - 111 \\ 512 + 012 \\ \hline 401 \;/\; \overline{1332} \end{array}$$

$\times \frac{1}{2}$

$= 200\frac{1}{2} \;/\; \overline{1332}$

$= 200 \;/\; 500-1332$

$= 200 \;/\; \overline{832}$

$= 199 \;/\; 1000-832$

$= 199 \;/\; 168 = 199168$

PROBLEMS :

1. Solve the multiplication problem taking suitable base or sub-base and indicate in the problem.

79 × 75	93 × 49	54 × 64
74 × 42	4970×4970	257×253
154×194	187×534	4999×4798

2. Solve the following :

(i)	31 × 33	(ii)	50001 × 10749
(iii)	38 × 66	(iv)	596 × 498
(v)	989×986	(vi)	43 × 59
(vii)	1003×998	(viii)	253×242
(ix)	42 × 56	(x)	17 × 16
(xi)	43 × 41	(xii)	692 × 608
(xiii)	889 × 811		

3. Check the correctness of the results by Vedic method.

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CHAPTER 7

NIKHILAM COROLLARIES

7.1 Introduction : The previously illustrated examples in chapter 6, were all solved by using the Nikhilam Sutra.

However, there are a few more corollaries arising out of the Nikhilam Sutra discussed in the previous chapter. Some of these have been presented in chapter 9 of Pushp 1.

7.2 The Third Corollary :

In mathematical astronomy we frequently require the multiplication of multiplier digits consisting entirely of nines with other numbers (multiplicand). For multiplication of this type the subsutra Ekanyunena-Purvena (ऐकन्यूनेन पूर्वेण) is used (Sutra 14).

7 2·1 Case I : Multiply 6 × 9

According to the sutra the multiplicand has to be decreased by 1 (this gives the LHS) and for the R.H S subtract the L.H.S from the multiplier. (Get compliments of nines).

Therefore $6 \times 9 = (6-1)/9-(6-1) = 5/4 = 54$

Illustrations :

(i) $13 \times 99 = (13-1)/(99-12) = 12/87 = 1287$
(ii) $778 \times 999 = 777/222 = 777222$
(iii) $9863 \times 999 = 9862/0137$

Comment : While obtaining the R.H.S. of the answer,

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by subtracting the L.H.S. from the multiplier (9999) all the digits including zero must be written (9999 − 9862) = 0137.

(iv) 3728387 × 9999999 = 3728386/6271613
= 37283866271613.

7·2·2 Case II :

If the multiplicand is having a smaller-number of digits than the multiplier, the Sutra Ekanyunena is equally applicable. It is necessary to fill in the blanks on the left with the required number of zeros. The remaining process is unaltered.

Illustrations :

(i) 8 × 99 = 08 × 99 = 07/(99-07) = 07/92 = 792
(ii) 67 × 999 = 067 × 999 = 066/933 = 66933
(iii) 737 × 99999 = 00737 × 99999 = 00736/99263
(iv) 83 × 9999999 = 0000083 × 9999999 =
= 0000082/9999917 = 829999917

7·2·3 Case III : (a) If the multiplier consists of smaller number of nines following the above procedure, the answer is obtained as under.

Illustrations :

(i) 17 × 9 = 16/9 − 16 = 16/$\bar{7}$ = 15/10 − 7 = 15/3 = 153
(ii) 163 × 9 = 162/9 − 162

= 162/$\overline{15}^{\bar{3}}$ = 162 − 15/$\bar{3}$
= 147/$\bar{3}$ = 146/10 − 3 = 1467

Comments : This case yields vinculum on the R.H.S.

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7·2·4 Case III : (b) In case of multiplication of numbers with lesser number of nines, the whole process can be still simplified by following the steps given here.

778×99

(i) The raw L.H.S. is just one less than the previous number $\quad 777/$

(ii) RHS digits are compliments of LHS digits from the nines (and equal to number of the nine digits).

$777/\ \bar{7}22$

Here $9-7=2$; $9-7=2$

To get the final LHS,

(iii) The excess digits of LHS are simply subtracted from the raw LHS obtained in (i)

OR we may say that the excess digits are written in vinculum form and carried to LHS.

$$777+\bar{7}/22=77022$$

Illustrations :

(i) $652 \times 99 = 651/\bar{6}\ 48$

$= 651+\bar{6}/48 = 64548$

(ii) $3687 \times 999 = 3686/\bar{3}\ 313$

$= 3686+\bar{3}/313 = 3683313$

(iii) $257642 \times 9999 = 257641/\overline{25}\ 2358$

$= 257641+\overline{25}/2358$

$= 2576162358$

7·3 Multiplication of Three Numbers :

The superfast Vedic procedure of multiplication can be further extended to multiply three, four or even more numbers simultaneously. Of course, the procedure becomes just a bit longer for larger numbers. This is

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another very special and unique feature in the cap of the (निखिलं नवतश्चरमं दशतः) 'Nikhilam Navatas' hramam Das 'atah' Vedic Sutra (Numbers 2). The following few examples shall serve the purpose of clearly demonstrating the very simple, straight forward step by step procedure.

As we already know, 'Nikhilam sutra' means all from 9 and last from 10. (Refer Pushp-1)

7·3·1 Type—1

Let us take a simple example of multiplying $98 \times 97 \times 96$.

Step 1 : We write these numbers in the Vedic base system style (refer chapter-1, of Pushp-1). We get the deficiency of each number from the base by invoking the 'Nikhilam sutra'. Therefore, we get

(The base is 100)

$$\begin{array}{c} 98 - 02 \\ 97 - 03 \\ 96 - 04 \\ \hline / \quad / \end{array}$$

Step 2 : For three numbers the product shall have three parts. Therefore two inclined lines (/ /) are drawn to demarcate the three parts of the answer.

Step 3 : We vertically multiply the three deficit numbers and the resulting number is the right hand portion of the answer. In this case $(-02) \times (-03) \times (-04) = -24 = \overline{24}$ is the required right hand protion. This is written down in the right hand space.

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98 — 02
97 — 03
96 — 04

/ / $\overline{24}$

Step 4 : The middle protion of the answer is obtained as follows. We multiply two deficit numbers at a time by rotation The summation of these three small products gives the middle portion of our answer and is written within the two slashes.

In the present example, we get

$(-02\times-03)+(-03\times-04)+(-04\times-02)$
$=6+12+8=26$

98 — 02
97 — 03
96 — 04

/26/$\overline{24}$

Step 5 : The left hand portion of the resulting product is obtained by the algebraic addition of the first number with the deficiency of the other two numbers.

In this example the L.H.S. is $98+(-03)+(-04)=91$. The resulting number is written at the appropriate space

98 — 02
97 — 03
96 — 04

91 / 26 / $\overline{24}$

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NOTE :—The left hand portion of the prod ct can also be obtained in several alternative ways. This, to mention again is the unique facet of Vedic Mathematics and each approach provides an easy check.

(i) The second number if added to the deficit numbers of the first and third numbers, also yields the same result. In this example $97+(-02)+(-04)=91$.

(ii) The same result is obtainable by additing the third number to the deficiencies of the first and second. Here $96+(-02)+(-03)=91$.

(iii) The left hand portion can also be obtained by the algebraic addition of the deficiencies of the three numbers to the base.

Here $100+(-02)+(-03)+(-04)=91$.

(iv) The same result is also obtainable by adding the three numbers and then subtracting two times the base from this addition.

Therefore, $98+97+96-(2\times100)=91$.

Comment : Any one of the several alternatives can be used for obtaining the L H.S. portion of the products. Further any alternative approach can be used again to confirm the correctness of the L.H.S. already obtained.

Step 6 : Therefore, the answer obtained in one line by the simple mental arithmetical process is $9126\overline{24}$.

Since the result may frequently have the Vinculum portion, the normalisation may be done in the usual manner to get at the final total positive answer. It is worthwhile to mention again that the Nikhilam sutra'

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is once again employed locally for normalising the Vinculum portion of any number.

Therefore $912\overline{6}\overline{2}\overline{4}=912576$

Observation :

An important point to be observed, though already emphasized in the previous chapter on Nikhilam sutra, is that in each of the steps number 3 and 4 respectively, the number obtained had only two digit number. Since the base in this example is 100, which has two zeroes and as such two digit answers of step 3 and 4 create no problem.

7·3·2 Illustration :

$$\begin{array}{l} 988 - 012 \\ 996 - 004 \\ 995 - 005 \\ \hline 979/128/\overline{2}\overline{4}\overline{0} \quad = 979127\ 760 \end{array}$$

R.H.S. $=(-012)\times(-004)\times(-005)$
$=\overline{2}\overline{4}\overline{0}$

Middle $=(-012\times-004)+(-004\times-005)$
$+(-005\times-012)$
$=128$

L.H.S. $=988-004-005=979$

7·3·3 Type 2 :

In case of multiplication of numbers with larger deviations from the base, the product of the deviations as obtained in steps 3 and 4 above, shall have number

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of digits larger than the number of zeroes in the base, the excess digits are carried over to the next place in the usual manner. The following illustrations shall explain the process

Illustrations :

(i) $88 \times 96 \times 95$

$$\begin{array}{l} 88 - 12 \\ 96 - 4 \\ 95 - 5 \\ \hline 79/\ 28/\ \bar{4}\bar{0} = 79\ /28\ /\bar{4}\bar{0} = 80\ 26\bar{4}\bar{0} \\ \quad 1\ \ \bar{2} \qquad\qquad\quad 1\ \ \bar{2} \\ \quad = 802560 \end{array}$$

(ii) $85 \times 94 \times 97$

$$\begin{array}{l} 85 - 15 \\ 94 - 6 \\ 97 - 3 \\ \hline 76/\ 53/\ \bar{7}\bar{0} = 76\ /53\ /\bar{7}\bar{0} = 775\bar{1}\bar{7}\bar{0} \\ \quad 1\ \ \bar{2} \qquad\qquad\quad 1\ \ \bar{2} \\ \quad = 775030 \end{array}$$

7·34 Type 3 :

The previous two types of examples have once again clearly illustrated the point that number of digits both in the R.H.S. and middle portions of the result must be equal to the number of zeroes in the base. The surplus digits, if any were carried over to the higher position digit. However, if the multiplication of the deficit digits yields the product (as obtained in steps 3 and 4) result

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having digits lesser than the number of zeroes in the base, then we prefix the zeroes to the individual products. The following example will illustrate this process.

Illustration :

$997 \times 996 \times 995$

$$\begin{array}{l} 997 - 003 \\ 996 - 004 \\ 995 - 005 \\ \hline 988 \quad /047 \quad /\bar{0}\bar{6}\bar{0} \end{array} = 988046940$$

R.H.S. $=(-3)\times(-4)\times(-5)=-60$, has only two digits, therefore the R.H.S. portion of the product is obtained by prefixing one zero before -60 (as there are three zeroes in the base which is 1000).

R.H.S $=\bar{0}\bar{6}\bar{0}$

Similarly, middle $= (-3)\times(-4)+(-4)\times(-5)$
$+(-5)\times(-3)$

$= 47$ has also one deficit digit,

hence middle$=047$

7 3·5 Type 4 :

In all the three types of problems discussed above all the numbers were below the base and as such the deviations from the base were obtained with a negative sign.

If some of the numbers are larger than the base and the remaining smaller than the base, the step by step procedure as already explained remains just the same.

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The only point to be kept in mind is that all the multiplications of the deviations from the base are done with appropriate sign of the deviations from the base. It goes without saying that the deviations are positive for the numbers larger than the base, and negative for numbers below the base. The following illustration is self-explanatory.

Illustration :

$1012 \times 997 \times 1006$

$$\begin{array}{r} 1012 + 012 \\ 997 - 003 \\ 1006 + 006 \\ \hline 1015 \,/\, 018/\overline{2}\overline{1}\overline{6} \end{array} = 1015017784$$

74 Multiplication of Four Numbers :

The step by step procedure outlined in the previous section can be extended for the multiplication of four or even five numbers simultaneously.

Illustration :

$$\begin{array}{r} 998 - 002 \\ 996 - 004 \\ 995 - 005 \\ 1004 + 004 \\ \hline 993/00\overline{6}/112/\overline{1}\overline{6}\overline{0} \end{array} = 992994111840$$

R H S $= (-2) \times (-4) \times (-5) \times (4) = -160$

Middle $= (-2 \times -4 \times -5) + (-4 \times -5 \times 4) +$

(R.H.S.) $(-5 \times 4 \times -2) + (-2 \times -4 \times 4)$

$= -40 + 80 + 40 + 32$

$= 112$

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Middle $=(-2\times-4)+(-4\times-5)+(-5\times+4)+$
L.H S. $(4\times-2)+(-4\times4)+(-5\times-2)$
$=8+20-20-8-16+10$
$=-6$

L H.S. $=998+(-4-5+4)$
$=998-5=993$

75 Multiplication of Five Numbers :

The same step-by-step procedure outlined in the previous section can be most conveniently extended to multiply even five or more numbers. Once again the entire operation can be performed mentally and the result obtained in just two lines. The usual rules of carry over of the excess digit and prefixing of zeroes for the deficit digits shall be applied for each part of the resulting multiplication. Further details with a very large number of examples are covered in the correspondence course.

PROBLEMS :

Q 1. Perform the following multiplications. Indicate the Sutra or subsutra in each case.

17×99	59×96	568×9999
414×9999	511×99	717×99
767×999	49367×9999	58437×9999
999×9999	63×999	767×99999

Q. 2 Multiply the three numbers in one line by using Nikhilam method.

(i) $8\times7\times6$ (ii) $97\times95\times99$ (iii) $93\times95\times98$
(iv) $88\times95\times96$ (v) $995\times997\times999$

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(vi) $985 \times 994 \times 996$ (vii) $984 \times 992 \times 994$

Q. 3. Obtain the product in one line.

(i) $1003 \times 1015 \times 1005$ (ii) $1011 \times 995 \times 1006$

(iii) $996 \times 989 \times 1004$ (iv) $92 \times 85 \times 111$

(v) $996 \times 997 \times 1003$

Q. 4. Multiply the four numbers in one line.

(i) $13 \times 12 \times 9 \times 8$ (ii) $97 \times 95 \times 103 \times 105$

(iii) $9997 \times 9995 \times 9989 \times 1005$

Q. 5. Check the solution of above problems by extending the Vedic Multiplication Check method given in Pushp 1.

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CHAPTER 8

Multiplicatian Using Vinculum & Checking

8 1 Introduction : In Chapter 10 of Pushp 1 we had learnt the general method of multiplication using the Urdhva-Tiryagbhyam (ऊर्ध्वतिर्यग्भ्याम्) Sutra. This simple but powerful sutra has many fold applications. It means Vertically and Crosswise we simplify different number of columns during the process of application of this sutra. At each step we cross multiply and take the summation of different products This summation becomes bigger as the digits and the number of digits become bigger. However, the process is rather simplified if we first convert all the digits greater than 5 into the Vinculum (विनकुलम्) form. The product of the digits in Vinculum form follows simple algebraic rules. (refer chapter 1)

8·2 The Procedure : The step by step process has already been explained in details (Pushp 1, Chapter 10). The same simple steps are used after converting the bigger digits into the Vinculum form. The following two examples shall explain the point.

8·2·1 Illustration 1 :

Multiply 79 by 68

2	1	
.	.	U
.	.	L
7	9	
6	8	
	2	
7		

Step I : Simplify the first column
R1=U1×L1 = 8×9=72

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R1=72 is the extreme right part of the answer, consisting of two digits. Therefore, we write 2 below the first column and excess digit 7 below the next space.

Step II : Operation of the first column with the second column gives us the next part of the product.

$$R2 = U1 \times L2 + U2 \times L1$$
$$= 9 \times 6 + 7 \times 8 = 54 + 56 = 110$$

Once again we enter 0 at the next space and 11 below the subsequent spaces.

```
  7   9
  6   8
 ------
      02
 11   7
```

With this we complete the operation of the first column.

Step III : After operating the first column with all other columns we start eliminating the columns one by one.

Eliminating first column, we are left with only the second column. Therefore $R3 = U_2 \times L_2 = 7 \times 6 = 42$ is the last part of the answer. We enter this value and add the carry over digits to get the final answer.

```
  7   9
  6   8
 ------
  4 2 0 2
    1 1 7
 ------
  5 3 7 2
 ------
```


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Alternatively we may convert this number into the Vinculum form and then multiply.

$79 = 1\bar{2}\bar{1}$ and $68 = 1\bar{3}\bar{2}$

$$\begin{array}{ccc} 1 & \bar{2} & \bar{1} \\ 1 & \bar{3} & \bar{2} \\ \hline \multicolumn{3}{c}{3/7/2} \\ \hline \end{array}$$

R1 $= U_1 \times L_1 = \bar{1} \times \bar{2} = 2$

R2 $= U_1 \times L_2 + U_2 \times L_1 = \bar{1} \times \bar{3} + \bar{2} \times \bar{2}$
$= 3 + 4 = 7$

R3 $= U_1 \times L_3 + U_3 \times L_1 + U_2 \times L_2$
$= \bar{1} \times 1 + 1 \times \bar{2} + \bar{2} \times \bar{3}$
$= \bar{1} + \bar{2} + 6$
$= 3$

With this we complete the operation of the first column with the remaining columns. For subsequent parts of the answer we start eliminating the columns one by one Eliminating the first column we simplify the remaining columns,

Therefore, R4 $= U_2 \times L_3 + U_3 \times L_2$
$= \bar{2} \times 1 + 1 \times \bar{3}$
$= \bar{2} + \bar{3} = \bar{5}$

R5 $= U_3 \times L_3 = 1 \times 1 = 1$

$$\begin{array}{cccc} & 1 & \bar{2} & \bar{1} \\ & 1 & \bar{3} & \bar{2} \\ \hline \multicolumn{4}{c}{1/\bar{5}/3/\ 7/2} \\ \hline \end{array}$$

$= 1\bar{5}372 = 5372$

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Symbolically we can say that

$$R1 = \begin{matrix} 1 & \bar{2} \\ 1 & \bar{3} \end{matrix}\left[\begin{matrix} \bar{1} \\ \bar{2} \end{matrix}\right] \qquad R2 = \begin{matrix} 1 \\ 1 \end{matrix}\left[\begin{matrix} \bar{2} & \bar{1} \\ \bar{3} & \bar{2} \end{matrix}\right]$$

$$R3 = \left[\begin{matrix} 1 & \bar{2} & \bar{1} \\ 1 & \bar{3} & \bar{2} \end{matrix}\right] \qquad R4 = \left[\begin{matrix} 1 & \bar{2} \\ 1 & \bar{3} \end{matrix}\right]\begin{matrix} \bar{1} \\ \bar{2} \end{matrix}$$

$$R5 = \left[\begin{matrix} 1 \\ 1 \end{matrix}\right]\begin{matrix} \bar{2} & \bar{1} \\ \bar{3} & \bar{2} \end{matrix}$$

For each part we simplify the columns written within the brackets.

Notes :— 1. We clearly observe that the arithmetical work involved is greatly simplified by using Vinculum approach.

2. The whole process can be carried out mentally after a little practice.

8·2·2 Illustration 2 :

$$\begin{array}{r} 2\,8\,7\,8 \\ \times\ 2\,7\,9\,8 \\ \hline \end{array} = \begin{array}{r} 3\,\bar{1}\,\bar{2}\,\bar{2} \\ 3\,\bar{2}\,0\,\bar{2} \\ \hline 9/\bar{9}/\bar{4}/\bar{8}/6/4/4 \end{array}$$

$$= 8052644$$

R1 $= \bar{2} \times \bar{2} = 4$

R2 $= \bar{2} \times 0 + \bar{2} \times \bar{2} = 4$

R3 $= \bar{2} \times \bar{2} + \bar{1} \times \bar{2} + \bar{2} \times 0 = 6$

R4 $= \bar{2} \times 3 + 3 \times \bar{2} + \bar{1} \times 0 + \bar{2} \times \bar{2} = \bar{8}$

R5 $= \bar{2} \times 3 + 3 \times 0 + \bar{1} \times \bar{2} = \bar{4}$

R6 $= \bar{1} \times 3 + 3 \times \bar{2} = \bar{9}$

R7 $= 3 \times 3 = 9$

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The entire calculations can be performed through mental process

8·2·3 Illustration 3 :

Let us do another multiplication of 5 by 5 digits in one line by the mental process. The above process is applied directly after first putting the numbers in the Vinculum form.

```
    2 7 8 1 9          5 4 3 2 1
  × 2 1 8 9 8          . . . . .  U
 ———————————           . . . . .  L
 ———————————
```

First we write the numbers in Vinculum form

```
        3 2̄ 2̄ 2 1̄
        2 2 1̄ 0 2̄
    ———————————————
    6/2/1̄/1̄/2/2̄/0/5/4̄/2
    ———————————————
```

$=61\bar{1}2\bar{2}05\bar{4}2$

$=609180462$

Note :—$R5 = U_1 \times L_5 + U_5 \times L_1 + U_2 \times L_4 + U_4 \times L_2 + U_3 \times L_3 = \bar{2}$ etc.

8·3 Alternate procedure : The Urdhva Sutra can also be applied for computing the multiplication results starting from left side and moving towards the right. However in this case it shall be essential to compute the product in two steps, as the carry over shall have to be added in the second step. The following example shall illustrate the steps.

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8 3·1 Illustration 4:

$$\begin{array}{cccc} 2 & 7 & 9 & 2 \\ 1 & 9 & 7 & 8 \\ \hline 3 & \bar{2} & \bar{1} & 2 \\ 2 & 0 & \bar{2} & \bar{2} \\ \hline \end{array} \quad \text{in Vinculum form} \qquad \begin{array}{cccc} 4 & 3 & 2 & 1 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array} \begin{array}{c} \\ U \\ L \end{array}$$

$6/\bar{4}/\bar{8}/2$

Extreme left result LR1 $= U_4 \times L_4 = 3 \times 2 = 6$

Similarly LR2 $= U_3 \times L_4 + U_4 \times L_3 = \bar{2} \times 2 + 3 \times 0 = \bar{4}$

LR3 $= U_2 \times L_4 + U_4 \times L_2 + U_3 \times L_3$
$= \bar{1} \times 2 + 3 \times \bar{2} + \bar{2} \times 0 = \bar{2} + \bar{6} + 0 = \bar{8}$

LR4 $= U_1 \times L_4 + U_4 \times L_1 + U_2 \times L_3 + U_3 \times L_2$
$= 2 \times 2 + 3 \times \bar{2} + \bar{1} \times 0 + \bar{2} \times \bar{2}$
$= 2$

With this we complete the operation of the fourth column with all others. Now the elimination process is done.

LR5 $= U_1 \times L_3 + U_3 \times L_1 + U \times L_2$
(Eliminating 4th column)
$= 2 \times 0 + \bar{2} \times \bar{2} + \bar{1} \times \bar{2} = 6$

LR6 $= U_1 \times L + U \times L_1$
$= 2 \times \bar{2} + \bar{2} \times \bar{1} = \bar{2}$ (Eliminating 3rd column etc.)

LR7 $= U_1 \times L_1 = 2 \times \bar{2} = \bar{4}$

$$\begin{array}{cccc} 3 & \bar{2} & \bar{1} & 2 \\ 2 & 0 & \bar{2} & \bar{2} \\ \hline \end{array}$$

$6/\bar{4}/\bar{8}/2/6/\bar{2}/\bar{4}$

$= 6\bar{4}\bar{8}26\bar{2}\bar{4} = 5522576$ Ans.

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8·4 Multiplication Check : The checking method for multiplication of numbers in the Vinculum form is very much similar to the Vedic check method for multiplication of ordinary numbers (refer Chapter II, Pushp 1). The simple steps are elaborated in the following sections. The only new point is about the turning point in Vinculum form as already explained in chapter 3, on checking of Vinculum number operations.

8 4 1 Illustration 5 : $1\bar{2}\bar{1} \times 1\bar{3}\bar{2} = 5372$ Check :

Step I : Write down the multiplicand, multiplier and the result in three rows in the usual manner and draw a big square in front of it. Draw two diagonals, thereby dividing the inner space into 4 triangles.

$$\begin{array}{r} 1\ \bar{2}\ \bar{1} \\ 1\ \bar{3}\ \bar{2} \\ \hline 5372 \\ \hline \end{array}$$

Step II : Add the digits of the top number. If the addition exceeds 9, once again add the digits of the results of addition Repeat this process till the final number is a single digit number.

In this example $1+\bar{2}+\bar{1}=\bar{2}$ is the resulting single number. Write this number in the top triangle.

Step III : Add the digits of the lower number i e. the multiplier. If the addition exceeds 9, once again add the digits of the result of this addition. Repeat this process till a single digit number is obtained as in previous step II.

In the present example $1+\bar{3}+\bar{2}=\bar{4}$, the resulting single number has been obtained in a single step.

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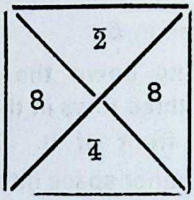
Write this number ($\bar{4}$) in the bottom triangle.

Step IV : Multiply the single digit number in the top triangle with the single digit number in the bottom triangle. Write the result adjacent to the right hand side triangle. If the result is of more than one digit add the digits of the resulting number. Write down this single digit number in the right hand side triangle.

$$\begin{array}{r} 1\ \bar{2}\ \bar{1} \\ 1\ \bar{3}\ \bar{2} \\ \hline 5\ 3\ 7\ 2 \\ \hline \end{array}$$



$\bar{2} \times \bar{4} = 8$

Step V : Add the digits of the total result of the multiplication. Once again if the addition exceeds 9, add these digits of the result of previous addition. Repeat this process till the final number is a single digit number. Write this single digit number in the empty space in left hand side triangle

Check :

If the number on the left hand side triangle is equal to the number in the right hand side triangle, we can be reasonably sure that the multiplication result is correct.

In the present example, we get $5+3=8$; $8+7=15$, $1+5=6$; $6+2=8$.

We observe that the numbers in the right hand side triangle and the left hand side triangle are both equal to 8. This confirms the correctness of the multiplication for any random errors.

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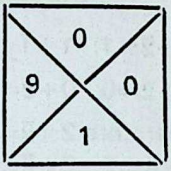
8 4 2 *Illustration 6* :

Check the solution of illustration 3.

Step (i) : Write the numbers and the product and draw the square.

$$\begin{array}{r} 3\ \bar{2}\ \bar{2}\ 2\ \bar{1} \\ 2\ 2\ \bar{1}\ 0\ \bar{2} \\ \hline 609180462 \\ \hline \end{array}$$



$0 \times 1 = 0$

Step (ii) : The number gives $3+\bar{2}=1$; $1+\bar{2}=\bar{1}$; $\bar{1}+2=1$; $1+\bar{1}=0$. This we write in the upper triangle.

Step (iii) : The multiplier gives $2+2=4$; $4+\bar{1}=3$; $3+\bar{2}=1$.

This we enter in the lower triangle.

Step. (iv) : The multiplication of the single digit numbers in the upper and lower triangles gives $0 \times 1 = 0$ which is entered in the RHS triangle.

Step (v) : The result gives $6+1=7$; (we ignore 9 and 0) $7+8=15$, $1+5=6$; $6+4=10$, $1+0=1$; $1+6=7$; $7+2=9$. But as already explained turning point in the checking of Vinculum numbers is 9, we know 9 is equivalent to 0; $8=-1$ (as $9-1=8$); $7=-2$ etc.

Check :

Hence the Vedic check for random errors has been satisfied as we have got equivalent numbers ($9=-0$) in the LHS and RHS triangles.

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8·4·3 Illustration 7 : Similarly, for illustration 4 :

Step (i) :

$$\begin{array}{r} 3\ \bar{2}\ \bar{1}\ 2 \\ 2\ 0\ \bar{2}\ \bar{2} \\ \hline 6\ \bar{4}\ \bar{8}\ 2\ 6\ \bar{2}\ \bar{4} \\ \hline \end{array}$$

Step (ii) : $3+\bar{2}=1;\ 1+\bar{1}=0;\ 0+2=2.$

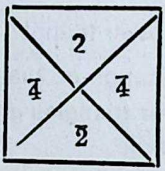
Step (iii) : $2+\bar{2}=0;\ 0+\bar{2}=\bar{2}$

Step (iv) : Therefore $2\times\bar{2}=\bar{4}$

Step (v) : $6+\bar{4}=2;\ 2+\bar{8}=\bar{6};\ \bar{6}+2=\bar{4};\ \bar{4}+6=2;$
$2+\bar{2}=0;\ 0+\bar{4}=\bar{4}.$ L.H.S.

Check is satisfied.

Note : We may also check the normal numbers.



$2\times\bar{2}=4$

$\bar{4}=\bar{4}$ O. K.

PROBLEMS :

Q. 1. Solve the following problems by the general method, alongwith other applicable methods. Compare and find out the most convenient sutra in each case.

(A) (i) 133×179 (ii) 141×281
(iii) 371×433 (iv) 1278×9034

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(v) 12078×10304 (vi) 378^2

(vii) 81265×32112

(B) (i) 89588×787 (ii) 8858×78

(iii) 6548×789 (iv) 432008×1212

(v) 9238×1010101 (vi) 987×987

(vii) 1021×1021

Q. 2. Solve following using Vinculum approach by general Sutra :—

(i) 787×391 (ii) 282×132

(iii) 181×98 (iv) 271×87

(v) 591×187 (vi) 483×179

(vii) 1892×197 (viii) 9867×918

(ix) 90908×81826 (x) 81739×9019

(xi) 906108×9079

Q. 3. Apply the superfast Vedic check method to solutions of Q1+Q2.

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CHAPTER 9

Division--By Nikhilam Method

9·1 Introduction : We have seen how NIKHILAM (निखिलं) Method can be used for the multiplication of big numbers. Some other applications of the NIKHILAM SUTRA in VINCULUM Numbers, Table etc have also been discussed. It is interesting to learn that the Nikhilam Sutra can also be used for the division. Like multiplication the Nikhilam Method is most convenient when the divisor consists of the big digits easier is the solution by NIKHILAM Method.

9 2 The NIKHILAM METHOD : As a first step we modify the divisor by using the NIKHILAM SUTRA (निखिलं नवतश्चरमं दशतः) and than use the modified divisor for subsequent simple operations which never involve any type of division. The step by step procedure is further illustrated with the help of following illustrations.

Type—1 :

9·2·1 Illustration 1 :

Divide 34 by 9

9 ÷ 34

Step 1 : We modify our divisor by using Nikhilam Sutra, Namely, the last digit is subtracted from 10 and next from nines. The modified divisor so obtained will be used for subsequent operation.

9 ÷ 34
1

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Step 2 : We write modified divisor (MD) and dividend (D) as follows

9 ÷ 34
1

Now we draw a slash in the dividend (D) after the number of digits equal to the number of the digits in the divisor. Here MD and D digits are numbers from LHS and RHS respectively. Since in this example we have only 1 digit divisor. We shall place an oblique line just before 1 digit of D.

1st 2nd 1st
9 ÷ 3 / 4
1

Step 3 : Now multiply the last digit of D (i e. appearing at 2nd column in the present case) with the last digit of MD (i e the one at the first and last place in the present case) and place the product just below the previous column of D (which is the 1st column in this example).

9 ÷ 3 / 4
1 3

In the present example we have already reached the first digit of the dividend and hence no subsequent operations of MD are required.

Step 4 : Now draw a line below the second row and sum the digits (column wise) appearing below 1st column, 2nd column and so on. The answer so obtained will have two parts. First part on R.H S. of oblique line, is

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Remainder R. The second appearing along L H S of oblique line, is the Quotient.

```
9 ÷ 3/4
1     3
-----
      3/7
```

Here 3 is Quotient and
7 is Remainder.

9·2 2 Illustration 2:

Divide 60 by 9

9 ÷ 60

Step 1 : Using NIKHILAM SUTRA we modify our divisor. Here MD = 1

Step 2 : We write MD and D as

```
9 ÷ 60
1
```

Step 3 : We draw a slash before 1 digit of D and number MD and D

```
    2nd 1st
9 ÷ 6 / 0
1
```

Step 4 : Multiply the last digit of D with the digit of MD and product is placed below the previous digit (1st column)

```
    2nd 1st
9 ÷ 6 / 0
1     6
```


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Step 5 : No subsequent operation is required.

Step 6 : Draw the line below the second row and add the 1st and 2nd column digits, we get result.

```
        2nd 1st
9 ÷ 6 / 0
        6
      ------
      6 / 6
```

Hence

Quotient = 6

Remainder = 6

9 2·3 Illustration 3 : Divide 71 by 9

9 ÷ 71

Straightaway we can write the solution followingly previously elaborated steps mentally.

```
9 ÷ 7 / 1
1     7
    -----
    7 / 8
```

Therefore R = 8

Q = 7

9·2 4 Illustration 4 :

Divide 1120211 by 9

```
9 ÷ 1 1 2 0 2 1 / 1
1     1
        2
          4
            4
              6
                / 7
-------------------
  1 2 4 4 6 7 / 8
```


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Hence Q = 1 2 4 4 6 7
R = 8

Notes :—(1) In all these division problems we have obtained the result without doing any division what so ever (2) We have only been multiplying a single digit by another single digit, and adding to get the result (3) In the previous examples since our MD was 1, there was really no multiplication at all.

9·3 Type 2 :

We shall now consider the second type of problems.

9·3·1 Illustration 5 :

Divide 1012 by 8

8 ÷ 1 0 1 2

Step 1 : Using NIKHILAM SUTRA we modify the divisor as MD = 2 (here)

Step 2 : We write as 8 ÷ 1 0 1 2
2

Step 3 : We draw a slash before 1 digit of D and number the digits of D from R.H.S.

4th 3rd 2nd 1st
8 ÷ 1 0 1 / 2
2

Step 4 : Multiply the last digit of D i e 1 with the MD digit 2 and product (1×2 = 2) is placed below the previous digit (3rd column), as before.

8 ÷ 1 0 1 / 2
2 2
———
1 2

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Step 4 (a) : On adding the digits of 3rd column we get a new operator as $0+2=2$ (operator)

Step 5 : Multiply this operator i.e. 2 with MD digit i.e. 2 and product $(2\times 2=4)$ is placed below the previous digit (2nd column).

```
8 ÷ 1 0 1/2
2     2
        4
  ---------
  1 2 5/
```

Step 5 (a) : Again adding the digits of 2nd column we get next operator as $1+4=5$ (operator).

Step 6 : Further multiply this operator (5) with MD digit (2) and product is placed below the previous digit (1st column).

```
8 ÷ 1 0 1/2
2       2 4/10
  ---------
  1 2 5/
```

Step 6 (a) : Further adding the digits of 1st column we reach the stage where we find that the remainder is bigger than the divisor.

```
8 ÷ 1 0 1/2
2       2 4/10
  ---------
  1 2 5/12
```

Note :—We find that the remainder so obtained is bigger than the divisor (8). Therefore we have to modify the remainder and carry the additional 1 to the quotient side.

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So that 12−8=4 is the final remainder and 125+1=126 is the final quotient. This can also be done mechanically by repeating the same NIKHILAM METHOD and this time we divide the Remainder as shown below :

```
8 ÷ 1 0 1 /2
2     2
        4
          10
  ----------
    1 2 5/1/2
            2
  ----------
    1 2 5/1/4
```

OR

126/4

Therefore Q = 126
R = 4

9 3·2 Illustration 6 : Divide 1011638 by 9

```
9 ÷ 1 0 1 1 6 3 / 8
1     1 1 2 3 9 /12
  ----------------
  1 1 2 3 9 12/ 20
            1
            /2/0
              2
  ----------------
            /2/2
  ----------------
1 1 2 4 0 2/2/2
```

OR 1 1 2 4 0 4/2

The Remainder is 2 and Q=112404

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Notes —:(1) All the rows have been converged into single row in order to save space.

(2) Here in 1st instance, we get remainder as 20 which is still greater than divisor Hence division has been further extended and the resulting quotient digit i.e. 2 has been added to previous quotient.

9 4 Type 3 : If divisor has more than one digit even then Nikhilam method works with equal ease.

9 4 1 Illustration 7 :

Divide 3 4 5 6 7 by 89

89 ÷ 3 4 5 6 7

Step 1 : We modify the divisor using NIKHILAM

MD = 11

Step 2 : We write as

89 ÷ 3 4 5 6 7

Step 3 : We draw a slash before 2 digits of D (from R H S) and number the digits of D from R.H S. respectively and number the digits of divisor from L.H.S. as

```
                 5th 4th 3rd 2nd 1st
 8 9  ÷   3  4  5 / 6   7
 1 1
1st 2nd  ————————————————
          3
```

Step 4 : Now multiply the last digit of D i e. 3 with 1st and 2nd digits of MD respectively (i e. 1 and 1) and the product is placed below the previous digits (4th and 3rd column respectively) as

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```
8 9 ÷ 3 4 5/6 7
1 1     3 3
---------------
3
```

Step 4 (a) : On adding the digits of 4th column we get the new operator (here 4+3=7).

Step 5 : Multiply this operator 7 with the MD digits 1 and 1 respectively and place the product below the 3rd and 2nd columns respectively

```
8 9 ÷ 3 4 5'67
1 1     3 3
          7/7
---------------
  3 7 15/
```

Step 5 (a) : After adding the digits of 3rd column we get a next operator i e 5+3+7=15.

Step 6 : Further multiply the operator 15 with MD digits 1 and 1 respectively and place the product below the 2nd and 1st column respectively.

```
8 9 ÷ 3 4 5 /6 7
1 1     3 3
          7/ 7
           /15 15
---------------
  3 7 15/
```

Step 6 (a) : Further adding the digits of all the columns

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```
8 9 ÷ 3 4 5 / 6 7
1 1     3 3
          7 / 7
            /15 15
  ---------------
  3 7   5/ 8  2
      1  2  2
  3 8   5/3 /02
1 1             33
  ---------------
  3 8   5/3 / 35
  3 8   8/35
```

Hence Q=388
R=35

9·4 2 Illustration 8 : Divide 20135 by 9818

```
9 8 1 8 ÷ 2/0 1 3 5
0 1 8 2     0 216 4
          ----------
          2/0 3 9 9
                1
     OR   2/0 4 9 9
```

Here the M.D contains 8 which is a bigger digit It could be modified by using Vinculum approach. However the products will also appear with negative sign and their subsequent addition will be algebraically done as explained below.

```
9 8 1 8 ÷ 2/0 1 3 5
0 1 8 2     0 4 4̄ 4
0 2 2̄ 2
        ----------
        2/0 5 1̄ 9
```


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On Normalising 2/0499. Hence Q=2
H=499

Note :—The operation becomes still simpler by using the Vinculum approach.

Note :—Zeros in M.D. are never ignored whenever these appear.

9 4 3 Illustration 9 :

Divide 98564318 by 9886

Here some of the dividend digits are greater than 5. Hence the dividend is modified by Vinculum approach. Thus

```
9 9 8 6 ÷ 9 8 5 6 4 3 1 8
9 9 8 6 ÷ 1 0 1̄ 4̄ 4̄/4 3 2 2̄
0 0 1 4     0 0 1 4
              0 0 0 0
                0 0 1̄ 4̄
                  0 0 3̄ 1̄2̄
                    0 0 0 0
              ----------------
              1 0̄ 1̄ 3̄ 0/ 3 4̄ 1̄0̄ 2̄
        OR    1 0 1̄ 3̄ 0/ 3 5̄ 0 2̄
OR  9 8 7 0/2 4 9 8
    Q = 9870      R = 2498
```

Comment : It may be noted that $\bar{1}\bar{2}$ in 4th row obtained by multiplying $\bar{3}$ by 4 is placed in 2nd column of dividend while summing $\bar{2}$ of $\bar{1}\bar{2}$ is added in 2nd column and remaining $\bar{1}$ of $\bar{1}\bar{2}$ is carried over to next column.

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Notes : (1) The operations of multiplications may yield more than one digit In that case entire product is to be placed in the proper position, columnwise as above. No shifting of columns is permissible in such cases.

(2) While summing columnwise, carryovers are possible But at no stage carry over from remainder side is to be done to the quotient side In such cases the remainder digits may be more than the columns appearing before oblique line.

9.5 Discussions :

(1) In all these examples we have seen that there is absolutely no need of division at all. As such the use of the trial digits as require in the present division process is eliminated.

(2) The whole process of division by even real big numbers (see illustrations (7, 8 and 9) is converted into simple series of multiplications and simple additions. Even these multiplications are of one digit number by another single digit, at a time.

(3) The work tends to become a little more difficult if the dividend has bigger digits but this difficulty can also be reduced by using the VINCULUM APPROACH (as in illustration 9).

(4) Infact in the Vedic method of division by NIKHILAM SUTRA we perform no division, no subtraction and use only small single digit multiplication and addition which is much simpler than the present system.

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9·6 Decimals : The NIKHILAM Method can further be extended even for computing the values in decimal places with equally ease and simplicity of procedure. There is almost nothing new in the procedure and methodology. The following illustrations shall be sufficient to explain this.

9·6·1 Illustration 10 :

Divide 345675 by 9 upto 10 decimal places

```
9 ÷ 3 4 5 6 7 / 5
1     3
        7
          12
            18
              25
---------------------
  3 7 2 8 5 /30
      1 1 2
  38405/3/0
            3
---------------------
  38405/3/3
```

38405+3/3

```
38408/3000000000/0
        333333333/3
-------------------
38408/3333333333/3
```

Hence Q = 38408 3333333333

9·6·2 Illustration 11 :

Divide 2341 by 89 upto 3 decimal place

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```
8 9 ÷ 2 3 / 4 1
1 1     2  2
          5 5
-----------------
  2 5 / 1 /16
          / 11
-----------------
  2 5 / 1/27
  26 / 2 7 0/ 0 0
         2 2
           9 9
             11 11
-----------------
  26 / 2 9 1/20  1
         1     1        =26/301/2/11
                                 22
                        ----------
                          26/301/2/33
                         =26.303/33
```

So Q = 26.303

Further a large number of additional problems have been solved as a part to the carrespondance course lessons.

PROBLEMS :

Q. 1. Perform division by Nikhilam Method in the following cases.

(i) 12483 ÷ 7988 (ii) 10015 ÷ 89

(iii) 210011 ÷ 8996 (iv) 1010108 ÷ 8988

Q 2 (i) Divide 1987 by 899

(ii) Divide 1294468 by 89997

(iii) Divide 2645678 by 7777

Q 3. (i) Divide 145672 by 99 upto 3 places of decimals.

(ii) Divide 1758011 by 887 upto 4 places of decimals.

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CHAPTER 10

Division By Paravartaya Method

10.1 Introduction : We have seen how Nikhilam Method can be applied for division by real big digits but however this is not at all suitable for division by smaller number. For division by smaller numbers, an equally easy and superfast Vedic method is available. This is based upon the PARAVARTAYA SUTRA. This is the fourth Sutra in the total list of sixteen and reads as 'परावर्त्य योजयेत्' Paravartaya Yojyet.

10 2 Divisors with Unity as First Digit :

The procedure followed in such cases is exactly the same as we have seen in the case of division by Nikhilam method The difference is only in initial steps, corresponding to the modification of divisor digits (MD). The stepwise process is illustrated in following examples.

10·2 1 Illustration 1 :

Divide 1235 by 112

112 ÷ 1235

Step 1 : We shall tabulate the divisor and dividend digits as follows :

(divisor digits) 112÷1235 (dividend digits 4-numbers)
3-numbers

Step 2 : (i) As stated earlier, the divisor is such that its first digit is one, while modifying the divisor ignore

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this digit. Thus modified divisor (MD) will contain only two digits (here they are 1 2).

(ii) Another modification in divisor is that all the remaining digits are taken down as digits of MD but with their sign reversed in every case. Thus here MD is $1\bar{2}$. In other words we transpose and apply as per Paravartaya Sutra.

(iii) Now MD contains two digits only, as such the oblique line shall be placed before two columns of dividend. As

Columns						
3rd	2nd	1st		4th	3rd 2nd 1st	
1	1	2	÷	1	2 / 3 5	
(M. D)	$\bar{1}$	$\bar{2}$				

Step 3 : All the remaining steps are the same as those already explained in the previous chapter of division by Nikhilam. Namely (i) complete the row of products obtained by multiplying with the first multiplier (the digit appearing in the 4th column of D is directly our first multiplier) by 2nd and 1st digit of MD and placing these below 3rd and 2nd columns of D respectively.

```
1 1 2 ÷ 1 2 / 3 5
  $\bar{1}$ $\bar{2}$     $\bar{1}$ $\bar{2}$
  ---------------
        1
```

(ii) The second row will be obtained by finding an operator multiplier by summing the digits appearing the previous column of D, here 3rd, and multiplying the same with 2nd and 1st digits of MD, and, placing these below 2nd and 1st column of D. Here operator is 1 ($2+\bar{1}=2-1=1$).

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```
1 1 2 ÷ 1 2 / 3 5
 1̄ 2̄     1̄ 2̄
            1̄ 2̄
 ----------------
      1 1 /
```

Here all the operations of multiplications have been exhausted, since we have reached the first digit of D.

Note :—This time since digits of MD are with negative sign the product may also have negative sign. Hence proper care of sign of product must be taken in handling the vinculum numbers.

(iii) Draw oblique line just below the oblique line of D and algebrically sum the digits column wise appearing on both sides of the line. The summation on R.H S of oblique line will be the Remainder R and the summation on L H.S. will be quotient Q.

```
Here is   1 1 2 ÷ 1 2 / 3 5
                  1̄ 2̄
                      1̄ 2̄
                 ----------
                  1 1 / 0 3
```

Result — R = 03
Q = 11

10 2 2 Illustration 2 :

Divide 13291 by 1125

Step 1 : We shall tabulate the divisor and dividend digits as follow :—

1125 ÷ 13291

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Step 2 : The first digit divisor is 1 so we ignore it. Further using Paravartaya sutra our modified divisor is 125 with negative sign as $\bar{1}\bar{2}\bar{5}$.

(ii) Our MD contains three digits so the oblique line is placed before three columns of D.

```
                5th 4th  3rd 2nd 1st
1125 ÷ 1    3  / 2   9   1
1̄2̄5̄
```

Step 3 : The last digit of D is 1, this is our first multiplier. Multiply this with MD digits and place below the 4th, 3rd and 2nd digits.

Step 4 : For next operator we add the digits of 4th column. So we get 2 as new operator multiplier ($3+\bar{1}=2$).

```
1 1 2 5 ÷ 1 3 / 2 9 1
  1̄ 2̄ 5̄     1̄   2̄ 5̄
  ---------------------
          1 2
```

Again multiply the operator digit 2 with MD digits, and placing at proper place. Now we have reached the first digit. Hence this is our last step.

Step 5 : Now we add the digits of the last three columns to get the result.

```
1 1 2 5 ÷ 1 3 / 2 9 1
  1̄ 2̄ 5̄     1̄ / 2̄ 5̄
                2̄ 4̄ 1̄0̄
  ---------------------
          1 2 / 2̄ 0̄ 9̄ or 11/1125+2̄0̄9̄
```

or 11/916

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Note : (1) Here we are left with the negative remainder (in vinculum form) as such we reduce the quotient by 1 and carry the full divisor to the remainder side (here 1125) and algebraically add to the negative remainder to get the final R. This situation can arise quite often in division by Parayartaya method.

(2) However, it is interesting to observe that there is hardly any division being done in the whole process. Instead we just follow a series of simple digit by digit multiplications and algebraic additions (with proper sign of course).

10·2·3 Illustration 3 :

Divide 1211 by 112

```
          1 1 2 ÷ 1 2 / 1 1
M. D.     1̄ 2̄       1̄  2̄
                      1̄ 2̄
          ------------------
                  1 1 / 2̄ 1̄
```

Q = 11 R = $\bar{2}\bar{1}$

Once again here the remainder is all negative, which is not acceptable. Hence we reduce quotient by one and add 112 algebraically to the remainder. The modified quotient is 10 (11 − 1) and remainder is $112 + \bar{2}\bar{1} = 112 - 21 = 91$. Therefore Q=10, R=91.

10·2·4 Illustration 4 :

Divide 13254 by 1132

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$$
\begin{array}{rcl}
1\ 1\ 3\ 2 & \div & 1\ 3\ /\ 2\ 5\ 4 \\
\bar{1}\ \bar{3}\ \bar{2} & & \bar{1}\quad \bar{3}\ \bar{2} \\
 & & \quad\ \bar{2}\ \bar{6}\ \bar{4} \\
\hline
 & & 1\ 2\ /\ \bar{3}\ \bar{3}\ \bar{0}
\end{array}
$$

Normalise as before

or 11/802 (R=1132−330)

10·3 Division by Big Numbers :

Now let us once again take up the division by big numbers and try to perform the same by the Paravartaya method. But as first additional step we shall convert the big divisor digits to the vinculum form and then follow the same step by step method. The following examples shall illustrate the details.

10·3·1 Illustration 5 :

Divide 112 by 89

Here by using Vinculum, divisor can be modified as $89 = 1\bar{1}\bar{1}$. Now by Paravartaya sutra transpose of last two digits is taken.

$$
\begin{array}{rcl}
1\ \bar{1}\ \bar{1} & \div & 1\ /\ 1\ 2 \\
1\ 1 & & \quad\ 1\ 1 \\
\hline
 & & 1\ /\ 2\ 3
\end{array}
$$

Note : We have got the same MD, as in case of Nikhilam method (Refer Illustration 7, Chapter 9).

10·3·2 Illustration 6 :

Divide 40000 by 9808

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Here 9 8 0 8 = 1 0 $\bar{2}$ 1 $\bar{2}$

1 0 $\bar{2}$ 1 $\bar{2}$ ÷ 4 / 0 0 0 0

0 2 $\bar{1}$ 2 0 8 $\bar{4}$ 8

4 / 0 8 $\bar{4}$ 8 Normalising

OR 4/0768

1033 Illustration 7 :

Divide 81946 by 898

Here both divisor and dividend need to be modified by Vinculum approach.

8 9 8 = 1 $\bar{1}$ 0 $\bar{2}$

8 1 9 4 6 = 1 $\bar{2}$ 2 $\bar{1}$ 5 $\bar{4}$

1 $\bar{1}$ 0 $\bar{2}$ ÷ 1 $\bar{2}$ 2 / $\bar{1}$ 5 $\bar{4}$

1 0 2 1 0 2

$\bar{1}$ 0 $\bar{2}$

1 0 2

1 $\bar{1}$ 1 / 2 3 $\bar{2}$

or 91/228

Notes : A difficult problem indeed, but how easy it has become in the Vedic approach. All the big digits have gone. The whole operation is a smiling job.

10·4 Discussions : In the above examples we have seen how much of similarity exists in the NIKHILAM and the PARAVARTAYA METHOD. As a matter of fact if we critically observe we have got exactly the same modified divisors, even by using the Paravartaya Sutra.

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2. Further the operational ease and simplicity of the magic Vedic method continues to exist in both the Nikhilam as well as the Paravartaya method.

10 5 Procedure for Other Divisors : All the previous examples had first digit as a unity in the divisor. But if this condition is not satisfied, we just need a small additional step as explained in the following examples. Just like the multiplication we use the *Anurupena Sub-sutra* in these cases (refer chapter 6).

10·5·1 Illustration 8 :

Divide 1354 by 184

Here 184 can be written as $2\bar{2}4$. In order to apply Paravartaya Sutra (first digit must be unity). For this we divide this divisor by 2. Thus the new divisor becomes $1\bar{1}2$.

```
   1 8 4 ÷ 1 3 / 5 4
2/ 2 2̄ 4 ÷   1   2̄
   1 1̄ 2           4 8̄
                 ----------
     1 2̄     1 4 / 7 4̄
                 ----------
           2/ 1 4 / 7 4̄     =7/66
```

Note :—The further operations are just the same without any change.

Here since divisor has been divided by 2, as such the raw quotient is also divided by 2 in order to get the final quotient. However, nothing is done to remainder, which remains unchanged. In this process we have again used

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the same subsutra Anurupena which was used in the subbase cases of multiplication.

10·5·2 Illustration 9 :

Divide 1233 by 224

Here we sub divide divisor by 2 and then proceed

$$
\begin{array}{l}
2\,/\,2\,2\,4 \div 1\,2\,/\,3\,3 \\
\quad 1\,1\,2 \qquad \bar{1}\;\;\bar{2} \\
\quad \bar{1}\,\bar{2} \qquad\quad \bar{1}\;\;\bar{2} \\
\hline
2/\;1\,1\,/\,0\,1 \qquad = 5\tfrac{1}{2}\,/01 \\
\quad\;\; 5\,/\,113
\end{array}
$$

when the quotient (i. e. 11) is divided by subdivisor (i e. 2) it yields five and half. This means quotient is 5 and half of original divisor (i. e. half of 224=112) is to be transferred towards remainder side. Thus the remainder becomes 112+1=113.

10 5·3 Illustration 10 :

Divide 7684 by 672

Here if we divide divisor by 6 we get 112. Hence

$$
\begin{array}{l}
6\,/\,6\,7\,2 \div 1\,\bar{2}\,\bar{3}\,/\,\bar{2}\,4 \\
\quad 1\,1\,2 \qquad\quad \bar{1}\,\bar{2} \\
\quad \bar{1}\,\bar{2} \qquad\quad\;\; 3 \quad 6 \\
\qquad\qquad\qquad\quad 2\,4 \\
\hline
\qquad\quad 1\,\bar{3}\,\bar{2}\,/\,6\,8 \qquad \text{or} \quad 6/\,68/68 \text{ or } 11\tfrac{1}{3}/68
\end{array}
$$

As $\frac{1}{3}$ of 672 = 224

and R = 224+68=292

Hence 11/292, is the result.

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Alternatively, we could also proceed by multiplying divisor by 2. Thus

$$
\begin{array}{rl}
2 \times 672 \div & 1\bar{2} / \bar{3}\bar{2}4 \\
1344 & \bar{3} / \bar{4}\bar{4} \\
\bar{3}\bar{4}\bar{4} & 15\ 20\ 20 \\
\hline
 & 1\bar{5} / 10\bar{4}4 \\
 & \text{or } 5 / 964
\end{array}
$$

By Anurupena $Q = 5 \times 2 = 10$

Remainder (=964) is greater than divisor (i. e. 672) Herce corrected quotient and remainder will be 11 and 292 ($R = 964 - 672 = 292$ and $Q = 10 + 1 = 11$) respectively.

10 6 Discussions :

1. By judicious use of vinculum, submultiples or sub-divisors as the case may be, we find through examples given above, that all the problems of divisors can be easily performed using Paravartaya sutra. Hence it is versatile sutra for performing division.

2. Most of the examples given above establish that divisions by Nikhilam and by Paravartaya are equivalent. Divisions can be performed in number of other ways but most suitable ones are illustrated above.

3. It is interesting to mention that exactly the same process is applicable even for the division of algebraic expressions. However the details of Paravartaya method for algebraic division shall be presented in Pushp 3.

10·7 Decimal Form : The Paravartaya method can also be extended for getting the answer in decimal form with equal case and simplicity.

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10 7·1 Illustration 11 :

Divide 563478 by 112 upto 3 decimal places

112 ÷ 563478

112 ÷ 1 $\bar{4}$ $\bar{4}$ 3 5 / $\bar{2}$ $\bar{2}$

$\bar{1}\bar{2}$ $\bar{1}$ $\bar{2}$

5 10

1 2

$\bar{1}\bar{4}/\bar{2}\bar{8}$

/ 7 14

---

1 $\bar{5}$ $\bar{1}$ 14$\bar{7}$ / $\bar{2}\bar{3}$ 12 $=1\bar{5}1,\ 4\bar{7}/\bar{2}\bar{3},\ 2$

$1\bar{5}04\bar{7}/\bar{2}\bar{2}2=5033/\bar{2}/\bar{2}2$

MD=$\bar{1}\bar{2}$ /24

$/\bar{2}/06$

$=5033+\bar{2}/6$

$=5031/6$

We continue further

5 0 3 1 / 6 0 0 / 0

$\bar{6}$ $\bar{1}\bar{2}$

6 12

---

5 0 3 1 / 6 $\bar{6}$ $\bar{6}$ / 12

5 0 3 1 / 5 3 4 / 12

5 0 3 1 . 5 3 4 / 12

Therefore Q=5031.534

10·8 Discussion : In the last two chapters we have seen that almost every problem of division canbe solved by using the Nikhilam or Paravartaya method. However

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this may not be always very efficient. There is the general method of division, which is the crowning gem of Vedic Mathematics. The details of the general method shall be presented in Pushp 3. How ever one problem has been explained in chapter 12.

PROBLEMS :

Q. 1. Perform division by Paravartaya method in the following cases.

(i) 1235 ÷ 113 (ii) 1248 ÷ 161

(iii) 13291 ÷ 1125 (iv) 12351 ÷ 1012

Q 2. (i) Divide 21356 by 8818 upto 2 decimal places.

(ii) Divide 1245 by 182 upto 3 places of decimals.

(iii) Divide 2534 by 224 upto 4 places of decimals.

Q. 3. Perform division using Nikhilam Sutra.

(i) 5243256 ÷ 7998

(ii) 345678 ÷ 6007

(iii) 14587819 ÷ 11465

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CHAPTER 11

DIVISION CHECK

11·1 Introduction : The simple 'Vedic Mathematics' procedure of checking the correctness of the multiplication results can easily be extended to check the calculations of any division of numerical numbers.

Further, it is really interesting to observe that the extremely simple Vedic Method of checking the calculations of any numerical division can be easily extended even to the **algebraic division.** This process is equally applicable to the problems where **remainder** is left over.

11 2 Procedure : It shall be interesting to explain the rather simple, super fast and accurate checking procedure, with the help of an example.

11·2·1 Illustration 1 :

Divide 46956 by 65. This problem can be solved by using Paravartya method. How ever, following the straight division process, the result is obtained in two lines (refer Pushp 3). The quotient is obtained as 722 with a remainder of 26.

Step (i) : Subtract the remainder from the dividend (46956—26=46930) this is exactly divisible by the divisor 65 in this case). The result obtained is exactly divisible by the divisor.

Step (ii) : By simple observation we can conclude that if our answer of division is correct. The modified dividend

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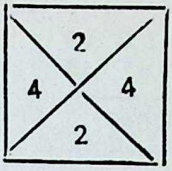
obtained in Step I should be equal to the multiplication of the quotient with the divisor.

Therefore

$$\begin{array}{r} 722 \\ \times\ 65 \\ \hline 46930 \\ \hline \end{array}$$



$2 \times 2 = 4$

Step (iii) : The problem at this stage has become exactly the same as has been detailed out in Multiplication Check (Chapter 8) :

Therefore the exact checking procedure can be straightaway applied.

(a) 7 2 2; $7+2+2=11$; $1+1=2$ (Upper triangle number)
6 5; $6+5=11$; $1+1=2$ (Lower triangle number)

Therefore the right hand side triangle number is $2 \times 2 = 4$.

(b) 4 6 9 3 0; $4+6=10$; $1+0=1$; $1+9=10$; $1+0=1$; $1+3=4$; $4+0=4$.

Therefore the left hand side triangle number is obtained as 4. The simple straight forward and superfast Vedic method of checking the division has been satisfied.

Step (iv) : $4=4$ Hence O. K.

11·2·2 Illustration 2 : Divide 876456 by 68

By applying the straight division process we get the quotient as 12889 with a remainder of 4.

Step 1 : Subtract the remainder from the dividend
($876456-4=876452$)

Step 2 : The modified dividend obtained in Step I should

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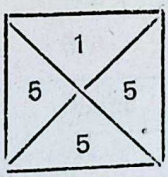
equal to the multiplication of the quotient with the divisor, i.e.

$$\begin{array}{r} 12889 \\ \times\ 68 \\ \hline 876452 \\ \hline \end{array}$$



$1 \times 5 = 5$

Step 3 : The problem at this stage has become exactly the same as has been detailed out in multiplication check.

(a) 12889; $1+2+8+8+9=28$, $2+8=10$, $1+0=1$;
(upper triangle number) $=1$

(b) 68; $6+8=14$, $1+4=5$
(lower triangle number)

Therefore the R.H.S. triangle number is $1 \times 5 = 5$.

(c) 876452; $8+7=15$, $1+5=6$;
$6+6=12$, $1+2=3$;
$3+4+5=12$, $1+2=3$;
$3+2=5$.

Therefore the L.H.S. triangle number is 5.

Step 4 : $5=5$. So our answer is correct.

11·3 Algebraic Division : The 'Vedic Mathematics' procedure outlined above for checking the division of numerical numbers is exactly applicable for the algebraic division as well. Only a small simple additional step is required for transforming the algebraic problem into a numerical problem. This is based on the Vedic Sutra (गुणित समुच्चय) Gunit Sammuchaya (Number 15). The details shall be presented in Pushp 3 of this Vedic Mathematics Pushp mala.

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CHAPTER 12

GLIMPSES

12·1 Introduction : The Ancient Vedic Mathematics is a vast subject and PUSHP 1 and 2 provide Vedic methods for Arithmetical operations. The applications in higher mathematics have already been revived in many areas. The pioneering book on Vedic Maths by Shankracharya Ji gives VM applications to algebraic operations, factorisations, simple quadratic and higher order equations, simultaneous quadratic equations, partial fractions, calculus, squaring, cubing, square root, cube root, coordinate geometry etc. etc. The recent research in U.K. includes applications of Vedic Mathematics to trignometry, three dimensional coordinate geometry, solution of plain and spherical triangles, linear and non-linear differential equations, matrices and determinants, log and exponentials etc.

In one of the recent five publications from U. K., Prof. K. WILLIAMS in his book, Discover Vedic Mathematics, has solved the complete mathematics papers of G. C. E. A-Level and O-Level examinations of U. K., there by demonstrating that the entire mathematics upto our 10+2 system can be done by using Vedic methods. The three English Professors in their recent publication Vertically and Crosswise (Refer list of new Books in VM News) have demonstrated how the Vedic 'URDHAVA SUTRA' alone can be used for solving a very large class of problems including evaluation of trignometrical

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functions and their inverse, logarithm, exponentials, higher order equations, partial differential equation and integral differential equation. Details of these are planned to be published in future PUSHPS of this PUSHPMALA. A few glimpses are presented in the following sections.

12·2 INTRODUCTION :

Straight Division : There is yet another method of division in Vedic style. This involves a bit more of mental working but the answer comes straight away in one step. The method is evolved by the combination of two Vedic sutras, namely Urdhav Tiryak (ऊर्ध्वनिर्यग्भ्याम) and Dhvajanka (ध्वजंक) (meaning on top of the flag).

Let us take one example in order to illustrate the new Dhvajanka method. The details shall be presented in Pushp 3.

12·2 1 Illustration 1 Divide 46959 by 65

65 ÷ 46956

Step 1 : Out of the divisor 65, we put down first digit (6) in the divisor column as operator and put other digit (5) on top of the flag (as flag digit) as shown below :

Flag Digit

5 ÷ 46956

(operator) 6

The entire division is to be done by 6.

Step 2 : As one digit has been put on the top we allot one place (at the right end of D) to the remainder portion of the answer and mark off from the other digits by vertical dots. Here last digit 6 has been marked off

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5 ÷ 4695 : 6 :
6

Step 3 : (i) We divide 46 (of dividend) by 6 and get 7 as the quotient and 4 as remainder, we put down 7 as the first quotient digit and prefix the remainder, (4) up before the next digit (i. e. 9) of the dividend. Then our actual gross dividend (G.D) is 49 for next step.

5 : 4695 : 6 :
6 4
———
7

(ii) from 49 we subtract the product of the flag digit (i. e. 5) and the first quotient digit (i. e. 7). So we get $49-7\times5=14$.

Thus 14 is our net dividend for further operation.

Step 4 : (i) Now divide this net dividend i.e. 14 by 6 we get 2 as second quotient digit and 2 as remainder which is placed before the next digit of dividend (i.e. 5). Now our Gross dividend is 25.

5 : 4695 : 6 :
6 42
———
72
———

(ii) Again from 25 we subtract the product of the flag digit (5) and the 2nd quotient digit (i.e. 2). So we get $25-(2\times5)=25-10=15$. This is the next net dividend.

Step 5 : (i) On dividing 15 by 6 we get 2 as third quotient digit and 3 as remainder to be placed before 6. Now 36 is our gross Remainder.

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(ii) Out of 36 we subtract the product of the flag digit (5) and the 3rd quotient digit (i.e. 2). So we get $36-(2\times5)$ $=36-10=26$. Here 26 is the remainder.

```
  5 : 4695 : 6 :
6       423
  ----------------
        722 : 26 :
```

Therefore Q=722
R= 26

12·3 Arithmetical Operations :

12·3·1 Express $\frac{17125}{59999}$ as a decimal upto 20 places.

By Vedic Sutra—Ekadhikena Purvena

$$\frac{17125}{59999}=.28542142369039483991$$

12.3.2 Solve the fraction $\frac{243}{1601}$

$$\frac{243}{1601}=.151780137414116179$$

12.3.3 Find square root .0009240160 upto 5 places of decimals.

```
6 : .00 : 09 : 2 4 0 1 6 0
  :     :    : 0 2 6 6
  ----------------------
  : .0  :  3 : 0 3 9 7 ....
```

12.3.4 Find the cube root of 33, 076, 161.
In three steps we get

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$33^1 \quad 076^1 \quad 161$

1

33 076 16

6

33 0761

Cube Root = 321

12.3 5 Curve fitting :—Find a quadratic passing through 3 points (8, 7) (6, 5) and (9, 4) changing origin to (6, 5), we get (2, 2), (0, 0), (3, −1),

If equation is $Y = AX^2 + BX$

By Urdhva Sutra $6\,Y = -8\,X^2 + 22\,X$

or $6\,y = -8\,x^2 + 118\,x - 390$

12.3.6 Evaluate Using Urdhva Sutra

$B = \exp(23.51)$

$B = \exp(23\ 51) = 1.62279 \times 10^{10}$

These are just a few stray examples and illustrations only.

12 4 Straight Division : Lef us take some more examples of the straight division method. (Refer 12·2 1)

12 4 1 Illustration 1 : Divide 5120 by 64

4 : 512 : 0 :

6

3

8 0 ; 0

The second net divident is 0 [32—8x 4]

12 4·2 Illustration 2 : Divide 7648 by 94

4 : 764 : 8 :

9

4 3

81 : 34 :

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12 4 3 In the previous examples we had seen that out of the two digits of divisor we are actually dividing by using one digit and the flag digit is being used for simple readjustment of the gross dividend Now let us take few examples of division by the 3/4 digit numbers.

12·4·4 Illustration 3 : Divide 24829685 by 2365

Step 1 : The Divisor can be written as

```
      65 : 2 4 8 2 9 6 : 8 5 :
   23
```

Here 23 is the operator and 65 is taken as Flag Digits. The entire Division is to be done by 23

Step 2 : Divide 24 by 23, we get quotient digit (i.e. 1) and the remainder (i e. 1). The remainder is placed before the next digit (i.e. 8) and the quotient is placed below the line

```
      6 5 : 2 4 8 2 9 6 : 8 5 :
  2 3         1
      ----------------------
        1
```

Our next gross dividend is 18.

(ii) Subtract the product of the quotient digit and the first flag digit from 18 (i e. 18—6=12) which is the net dividend at this stage

Step 3 : Divide the net dividend 12 by 23 which is smaller than the divisor. So 12 is placed before the next digit (2) as our remainder (of this step) and the quotient is 0.

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```
      6 5 : 2 4 8 2 9 6 : 8 5
2 3
            1 12
          ----------------------        65
            1 0                         10=5
```

So our next gross dividend is 122.

(ii) Now subtract the product of the 2 quotient digits (i.e. 10) and the 2 flag digits (i e. 65) by the Urdhava Tiryak Sutra) $65 \times 10 = 5$ (by cross product) 5, from 122 we get 117 (122—5=117) which is the next net dividend.

Step 4 : Again divide the net dividend (117) by 23 we get quotient digit (i.e. 4) and the remainder (25), place it before the next digit (i.e. 9)

```
      6 5 : 2 4 8 2 9 6 : 8 5
2 3
            1 12 25
          ----------------------
            1 0 4
          ----------------------
```

So our next digit is 259.

(ii) Again subtract the cross product of 65 (the flag digits) and the last two digits of quotient from 259 (259—(65×04=24)=235), the cross product $\begin{array}{r} 65 \\ \underline{04} \end{array} = 24$

So 235 is our net dividend.

Step 5 : (i) Further divide 235 by 23, we get 9 as quotient digit and the remainder 28 is placed before the next digit (i e. 6). (Refer item 12.4.7)

```
      6 5 : 2 4 8 2 9 6 : 8 5 :
2 3
            1 12 25 28
          ----------------------
          1 0 4 9
```


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Therefore 286 is our next gross dividend.

(ii) Further subtract the product of the flag digit (65) and the last two digits of the quotient (i e. 49) from 286 we get

$$286-(65\times 49=74) = 212 \text{ (net dividend)}$$

Step 6 : (i) Divide the net dividend 212 by 23 we get quotient digit 8 and the remainder 28, place it before the next digit (i e 8). Since we have reached the digit marked off as remainder portion (indicated : 85 :), we stop the division process.

```
      6 5 : 2 4 8 2 9   6 : 8 5
2 3
            1 12 25 28 28
          ----------------------
            1 0 4 9 8 :
```

Hence 288 is next gross remainder.

(ii) Subtract the product of flag digit (65) and the last two digits of the quotient (i.e 98) from 288. We get (288—(65×98=93)=195) 195 which is placed before the next digit 5 at this stage So the gross R is 1955.

Step 7 : To find the net remainder again subtract the product of the last flag digit (i e. 5) and the last quotient digit (i e 8) from 1955. We get (1955—(8×5)=40 =1915). So 1915 is the net remainder

```
      6 5 : 2 4 8 2 9 6 : 8    5 :
2 3
            1 12 25 28 28 195
          ----------------------
            1 0 4 98/1 9 1 5
```

Q = 1 0 4 9 8
R = 1 9 1 5

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Even these simple steps can all be performed just with mental working along with a very little rough work and the quotient and remainder obtained in a single step. Let us practice the process by doing the mental working in the following illustrations. Further in next few examples, it is more convenient to work with a single flag digit.

12·4·5 Illustration 4 : Divide 26786 by 114

```
1 1 4  ÷  2 6 7 8 6
    4  :  2 6 7 8 : 6
  1 1       4 6 12
            ----------
            2 3 4  : 1 1 0 :
```

Therefore Q = 2 3 4

R = 1 1 0

12·4·6 Illustration 5 : Divide 342945 by 136

```
    6 : 3 4 2 9 4 : 5
  1 3
          8 5 3 9
        ----------
        2 5 2 1 : 8 9 :
```

Therefore Q = 2521

R = 89

12 4 7 Special Cases : In this process of division sometimes we find that the gross remainder left over is too small the necessary adjustment product is bigger than it. This problem is easily handled by just using the next lower digit as our new quotient digit (Refer illustration 3). This trial and adjustment situation is equally encountered even in the division by the present method. The following example further illustrates this facet.

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12·4·8 Illustration 6 : Divide 601425 by 76

```
  6 : 6 0 1 4 2 : 5 :
7       11 6 3 5
  ----------------
      7 9 1 3 :3 7:
```

Here in the first division by 7 we can put down 8 as quotient digit, but the remainder then left (41) will be too small for the subtraction expected at the next step (48). So we take 7 as the quotient digit and prefix the remainder 11 to the next dividend digit and then proceed as usual.

12·4·9 Decimal form : The Vedic DHAVAJANKA METHOD can be easily extended to get the result in decimal form by continuing the process of division of the remainder. We place the additional zeros after the dividend. The following examples shall illustrate the process.

12·4·10 Illustration 7 :

Divide 8352 by 79 to three places of Decimals.

```
  9 : 8 3 5 : 2 0 0
7
      1 4 10 8 3
    ---------------
    1 0 5 : 7 2 1
    ---------------
```

The process of solving these questions is same. We

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put the decimal point as soon as we start the division of the remainder.

12·4·11 Illustration 8 :

Divide 711.023 by 38 to three places of decimals

```
 8 : 7 1 1. 0 2 3
3
      4 9 6 1 1
    ---------------
      1 8. 7 1 1
    ---------------
```

12·4·12 Illustration 9 :

Divide 221 by 52 to three places of decimals.

```
  2 : 2 2 1. 0 0
5       2 3 1
    ---------------
        4. 2 5 0
    ---------------
```

12·4·13 Illustration 10 :

Divide 7 5 by 54 to 5 places of decimals.

```
  4 : 7. 5 0 0 0 0
5       2 6 8 8 8
    ---------------
        . 138888
    ---------------
```


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12·5 Algebraic Expressions :

12.5.1 Solve equation : By Vedic sutra – Shunyam Samya Sammucaya.

$$\frac{1}{x+3} + \frac{1}{x-8} = \frac{1}{x+4} + \frac{1}{x-9}$$

Answer $x=5/2$

12 5.2 Factorize the equation $15x^2-14xy-8y^2$ using sutras Anurupena and Adyamadyenantyamentyena.

$15x^2-14xy-8y^2=(3x-4y)(5x+2y)$

12 5.3 Find the roots of the biquadratic equation

$x^4-20x^3+137x^2-382x+360=0$

using lopanasthapana sutra in four steps we get

$x=5,\ 4,\ 2$ or 9

12·6 Applications of Urdhava Sutra :

12.6.1 Determinants :

Evalutate $\begin{vmatrix} 2 & 6 & 7 \\ 5 & 4 & 8 \\ 3 & 1 & 9 \end{vmatrix}$

$-7 \quad -21 \quad 28 \qquad =-119$

Here Urdhava Sutra is used.

12.6.2 Solution of Simultaneous equation :

$$\begin{array}{r|lll|r} -8 & 4x-2y & +3z & =8 & -5 \\ 14 & 2x-3y & +z & =1 & -17 \\ 13 & 3x+2y & -z & =3 & 4 \end{array}$$

$\therefore D=33;\ XD=33;\ -YD=-33;\ ZD=66$

whence $x=y=1;\ z=2$ Urdhava Sutra (No. 3) is used.

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ANSWERS

Chapter 1 :

Q. 1 (i) 17644 (ii) 391409 (iii) 3209037

Q. 2 (i) —5642003 (ii) —185729

Q. 3 (i) $\overline{14}$ (ii) 27 (iii) $\overline{32}$ (iv) 34

Q. 4 (i) F (False) 301 (ii) F, 010 (iii) F

Q. 5 (i) 42314 (ii) 30001 (iii) 2006

Chapter 2 :

Q. 1 (i) 990 (ii) 19940 (iii) $2\overline{5}4\overline{6}1$

(iv) 62838 (v) 47988 (vi) 4466

(vii) $\overline{3}9\overline{2}$ (viii) 3018 (ix) 333

(x) 267 (xi) 1067

Chapter 3 :

Q. 1 (i) F (ii) F (iii) F (iv) F (v) F.

Chapter 4 :

Q. 1 (a) 3 hr. 31 min. 11 sec. (b) 15 yd. 0 ft. 8 inch

(c) 54 ton 26 kg 27 gm (d) 14 km 146 meter 80 cm

(e) 3 yr. 5 mth 9 day. 14 hr 7 minute.

Chapter 5 :

Q. 1 (i) 10 sq ft 100 sq inch (ii) 8 sq yd 8 sq ft

(iii) 9 sq yd 6 sq ft 45 sq inch

Q. 2 (i) 46 cu ft 1712 cu inch (ii) 48 cu ft 576 cu inch

Q. 3 (i) 3·5 sq ft 43·5 sq inch (ii) 11 sq ft 141 sq inch

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Chapter 6 :

Q. 1 (i) 5925 (ii) 4557 (iii) 3456
(iv) 3108 (v) 24700900 (vi) 65021
(vii) 29876 (viii) 99858 (ix) 23985202

Q 2 (i) 1023 (ii) 537460749 (iii) 2508
(iv) 296808 (v) 975154 (vi) 2537
(vii) 1000994 (viii) 61226 (ix) 2352
(x) 272 (xi) 1333 (xii) 420736
(xiii) 720979

Chapter 7 :

Q 1 (i) 1683 (ii) 5544 (iii) 27832
(iv) 206586 (v) 50589 (vi) 70983
(vii) 8758233 (viii) 493620633 (ix) 584311563
(x) 9989001 (xi) 62937 (xii) 76699233

Q 2 (i) 336 (ii) 912285 (iii) 865830
(iv) 802560 (v) 991022985 (vi) 975173640
(vii) 970271232

Q. 3 (i) 1023135225 (ii) 1011980670 (iii) 988984176
(iv) 868020 (v) 995991036

Q. 4 (i) 11232 (ii) 99660225
(iv) 9986000803499175

Chapter 8 :

Q. 1 (A) (i) 23807 (ii) 39621 (iii) 160643
(iv) 11545452 (v) 12445 712 (vi) 142884
(vii) 2609581680

(B) [i] 70505756 [ii] 690924 [iii] 5166372
[iv] 523593696 [v] 9331313038 [vi] 974169
[vii] 1042441

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Q. 2 [i] 307717 [ii] 37224 [iii] 17738
[iv] 23577 [v] 110517 [vi] 86457
[vii] 372724 [viii] 9057906 [ix] 7438638008
[x] 737204041 [xi] 8226554532

Chapter 9 :

Q. 1 [i] Q=1 R=4495 [ii] Q=112 R=47
[iii] Q=23 R=3103 [iv] Q=112 R=3452

Q. 2 [i] Q=2 R=189 [ii] Q=14 R=34510
[iii] Q=340 R=1498

Q. 3 [i] 1471 434 [ii] 1981 9740

Chapter 10 :

Q. 1 [i] Q=10 R=105 [ii] Q=7 R=121
[iii] Q=11 R=916 [iv] Q=12 R=207

Q. 2 [i] 2 42 [ii] 6·840 [iii] 11 3125

Q. 3 [i] Q=655 R=4566 [ii] Q=57 R=3279
[iii] Q=1272 R=4339

Chapter 11 :

Q. 1 (i) False (ii) False (iii) False (iv) True
(v) False

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ERRETA

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37	—	21	Corretness	Correctness
45	—	10	Vary	very
45	—	2nd last	yards	yard
50	—	1	squre	square
54	—	22	paraller	Parallel
55	—	19	inche	inch
56	—	2	puahp	pushp
57	—	4	Multipler	Multiplier
67	—	20	on the	—

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FUTURE OF VEDIC MATHS

Future Research : There are almost infinite number of applications of Vedic mathematics in different branches of science and technology. Prof. Andrew Nicholas of Polytechnic of North London forsees very bright future for Vedic Mathematics In addition to the topics already listed under the VM News, Revered Bharti Krishen Tirthji Maharaj has mentioned about the Vedic methods for solving problems of Interest, Statics, Dynamics, Hydrostatics, Pneumatics, Applied mechanics, Solid Geometry, Plain and spherical trigonometry and Astronomy etc etc. Dr Verma of Delhi has recently informed that Revered Swamiji had delivered a series of 12 lectures on Vedic Mathematics for Fluid dynamics aspects of Aero-nautics.

It seems that what is available today in Vedic mathematics is only a small visible tip above the water surface, of the huge ice berg floating in sea.

These require dedicated research work to rediscover the hidden treasures.

The feedback from those who have learnt Vedic Mathematics in a nutshell brings out that :—

1. VM converts the tedious mathematics into an interesting and playful subject, which children learn with smiles.

2. Participating teachers and children often ask why VM is not taught in every school and college.

3. The participants invariably derive a sense of national pride and 'Shradha' for our heritage.

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This task of developing and spreading Vedic mathematics amongst our countrymen requires the dedicated efforts of the lovers of Mother India who have 'shradha' for Bhartiya cultural heritage.

ANCIENT VEDIC MATHEMATICS

Pushp Mala :—With a view to provide the comprehensive details of this wonderful Ancient Bhartiya Cultural Heritage SSS is planning to publish the PUSHP MALA of Vedic Maths. PUSHP-1 and 2 have already been published in English and Hindi.

The SSS most prayerfully and humbly announces the planned publication of the PUSHP—3 in English and Hindi by middle of 1988. You can book your copy to avail of the prepublication concession of 20 precent by sending an advance of Rs. 10/- till the Baisakhi, April 13, 1988.

INVITATION

You are most cordially invited to participate in this Gyan Yagya by giving Ahuti of your precious time. You can do as many of the following as convenient.

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—Spreading the news of Vedic Mathematics.

- ☐ Chatting about VM when you are amongst unknown people.
- ☐ Giving small illustrations of the superfast cases to interested students, teachers and persons.
- ☐ Informing all that Vedic maths is a complete scientific system.
- ☐ Giving demonstration lectures in your near by institutions.
- ☐ Organising the short VM courses at your place.

—Spreading the Vedic Maths Literature.

- ☐ Gifting VM books to your kith and kin.
- ☐ Gifting VM literature to the libraries in your area.
- ☐ Giving VM books as prizes at the functions you organize.

—Developing Vedic Mathematics

- ☐ Collecting the ancient superfast computational procedures and literature and sending to/or/informing SSG.
- ☐ Applying Vedic Sutras to solve the problems of different sciences and sending to SSG for compilation and publication.
- ☐ Meditating on Vedic Sutras and further developing them and their applications.

SSG is meant to assist you in this Yagya. This activity according to Vedanta shall decidedly help us in cleaning our mind and then the SELF shall automatically stand revealed to the self.

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A few opinions about Ancient Vedic Mathematics :—

—"I see unlimited potential in the Vedi Mathematics System".

—Prof. K. Williams, Londo

—"It is tempting to think of them (Indians) a computer scientists.....Let us not forg that among the great accomplishments of the India thinkers were the invention of zero, and of the Binary number system (used in all computers) a thousand years before the West".

—Dr. R. Briggs of NASA, Ca
in Jl. of artificial Intelligence, Sp

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